Table of Contents

Internet Appendix I 1
Expected Profits for Each Type of Order .......................... 1

Internet Appendix II 16
Proof of Proposition 1 .................................................... 16
Proof of Lemma C.2 ...................................................... 21
Proof of Proposition 2 .................................................... 58

Internet Appendix III 63
Proof of Proposition 3 .................................................... 63
Proof of Proposition 4 .................................................... 69
Proof of Proposition 5 .................................................... 75
Proof of Proposition 6 .................................................... 86

Internet Appendix IV 93
Internet Appendix I (Expected Profits for Each Type of Order)

In this Appendix we explain how the expected profits of each possible type of order are calculated. Note that the expected profits for each rational trader depend on the information contained in the LOB and the information the trader has about the liquidation value of the asset. Note that the game is symmetric and, hence, we focus on symmetric equilibria.

Let us define $\Omega_o$ and $\Gamma_o$ as the probability that an informed trader and uninformed trader at $t = 1$ choose an order $O \in O_D$, where $o = 0$ corresponds to a $NT$ order; $o = 1$ to a $MO$; $o = 2$ to a $LO$; $o = 3$ to a $DO$; and such that $\sum_{o=0}^{3} \Omega_o = 1$ and $\sum_{o=0}^{3} \Gamma_o = 1$.

We also define as $B_t$ the set of all possible states of the LOB after a trader arrived at $t$ and eventually traded, and as $B_t \in B_t$ a possible state of the LOB. For example, $B_1$ is a possible state of the LOB such that

$$B_1 = \begin{cases} 
\varnothing, & \text{if the best prices in the LOB are } (A_1^1, B_1^1), \\
BMO, & \text{if the best prices in the LOB are } (A_1^2, B_1^1), \\
BLO, & \text{if the best prices in the LOB are } (A_1^1, B_1^1 + \tau), \\
SMO, & \text{if the best prices in the LOB are } (A_1^1, B_2^1), \\
SLO, & \text{if the best prices in the LOB are } (A_1^1 - \tau, B_1^1). 
\end{cases}$$

We also define $B_{2End}$ a possible state of the LOB at the end of the period $t = 2$.

In the second trading period the expected profits of each strategy depends on the state of the LOB (which on its turn depends on the chosen strategy at $t = 1$). Uniformed traders at $t = 2$ form beliefs about the strategies and type of player in $t = 1$. Thus, the uninformed traders’ belief at $t = 2$ about the probability that the $MO, LO$, and observed in the LOB was submitted by an informed trader as $X, Y$ and $Z$, respectively and are defined in the paper by ($B.1$), ($B.2$) and ($C.1$).

We start solving the model backwards at $t = 2$, the last date when a rational trader can decide where and how to trade. We consider at $t = 2$ all the possible states. There are 5 possible states depending on which type of trader arrived ($LT, IH, IL, UB, US$). Also, depending on which was the traders’ optimal choice at $t = 1$, there are five possible states of the LOB at the end of $t = 1$. In what follows we calculate the expected profits in each of the possible combinations of types of trader and states of the LOB. Notice, that in what follows we do not include the profits of choosing $NT$ since these are always null in all the possible states and times.

$t = 2$

2.1. State ($t = 2$, Liquidity trader)

A liquidity trader arrives with probability $1 - \lambda$. With probability $1/2$ he will choose a $BMO$ and with probability $1/2$ a $SMO$. 

1
2.2. State \((t = 2, \text{IH})\)

In this state, the liquidation value of the asset is \(v = v^H\), and an informed trader arrives. This trader is a buyer. The best prices of the \(\text{LOB}\) at the end of \(t = 1\) can take the following values: \((A_1^1, B_1^1)\), \((A_2^1, B_1^1)\), \((A_1^1, B_1^1 + \tau)\), \((A_1^1, B_1^2)\), or \((A_1^1 - \tau, B_1^1)\). Hence, we distinguish five possible states of the \(\text{LOB}\) at the beginning of the second trading period.

2.2.1. \((A_1^1, B_1^1)\)

This occurs when either a trader that arrives at the market at \(t = 1\) chooses to go to the \(\text{DP}\) or chooses \(\text{NT}\). Therefore, \(B_1 = \emptyset\). The expected profits for the various types of orders at \(t = 2\) are as follows:

\[
\mathbb{E}(\Pi_{BLO,2}^H|B_1 = \emptyset, v = v^H) = v^H - A_1^1 = (\kappa - k_1) \tau > 0, \quad \text{and} \\
\mathbb{E}(\Pi_{BDO,2}^H|B_1 = \emptyset, v = v^H) = \theta_2^0 \left( v^H - \frac{A_1^1 + B_1^1}{2} \right) = \theta_2^0 \kappa \tau \geq 0.
\]

For computing the expected profits of a \(\text{BLO}\), note that a \(\text{BLO}\) is chosen by an informed trader who observes \(v^H\) \((\text{IH})\) and arrives at the market at \(t = 2\) can only be executed if at \(t = 1\) there is an uninformed seller who chooses a \(\text{DO}\) and the order is not executed. Furthermore, given that the trader at \(t = 2\) observes \(v^H\), the possible cases such that the \(\text{LOB}\) has not changed during \(t = 1\) are the following: 1) an informed trader who observes \(v^H\) and who goes to the dark at \(t = 1\); 2) an uninformed buyer who goes to the dark at \(t = 1\); 3) an uninformed seller who goes to the dark at \(t = 1\); 4) an uninformed buyer who chooses \(\text{NT}\) at \(t = 1\); and 5) an uninformed seller who chooses \(\text{NT}\) at \(t = 1\).

Therefore, the probability of execution of a \(\text{BLO}\) chosen by an informed trader who observes \(v^H\) and faces \((A_1^1, B_1^1)\), which is denoted by \(p_{BLO,2}^H(B_1 = \emptyset)\), is given by

\[
p_{BLO,2}^H(B_1 = \emptyset) = \frac{(1 - \theta_2^1)^{1 - \frac{\pi_3}{2}} \Gamma_3}{\pi_3 + (1 - \pi) (\Gamma_0 + \Gamma_3)},
\]

and the corresponding expected profits are given by

\[
\mathbb{E}(\Pi_{BLO,2}^H|B_1 = \emptyset, v = v^H) = p_{BLO,2}^H(B_1 = \emptyset) \delta(v^H - B_1^1 - \tau) = p_{BLO,2}^H(B_1 = \emptyset) \delta (\kappa + k_1 - 1) \tau.
\]

Comparing profits, we conclude that the informed trader never chooses \(\text{NT}\) since this option is strictly dominated by a \(\text{MO}\).

2.2.2. \((A_2^1, B_1^1)\)

This state of the \(\text{LOB}\) occurs when at \(t = 1\) a trader arrives at the market and chooses a \(\text{BMO}\) (denoted by \(B_1 = \text{BMO}\)). In this case the \(\text{LOB}\) has changed in the ask side: \(A_1^2 = A_1^1\) and \(B_1^2 = B_1^1\).
The expected profits for the various types of orders at $t = 2$ are as follows:

$$
\mathbb{E}(\Pi_{BMO}^H | B_1 = BMO, v = v^H) = v^H - A_2^2 = (\kappa - k_2) \tau > 0,
$$

$$
\mathbb{E}(\Pi_{BDO}^H | B_1 = BMO, v = v^H) = \theta_2^I \left( v^H - \frac{A_2^2 + B_1^2}{2} \right) = \theta_2^I \left( \kappa - \frac{k_2 - k_1}{2} \right) \tau \geq 0, \text{ and}
$$

$$
\mathbb{E}(\Pi_{BLO}^H | B_1 = BMO, v = v^H) = 0,
$$

since the probability of execution of any LO at $t = 2$ is null. Comparing expected profits, we conclude that the informed trader does not choose a LO or NT since these options are strictly dominated by a MO.

### 2.2.3. $(A_1^1, B_1^1 + \tau)$

Note that the LOB has changed in the bid side because during the first trading period a trader has arrived at the market and has chosen a BLO (denoted by $B_1 = BLO$). The expected profits for the various types of orders at $t = 2$ are as follows:

$$
\mathbb{E}(\Pi_{BMO}^H | B_1 = BLO, v = v^H) = v^H - A_1^1 = (\kappa - k_1) \tau > 0,
$$

$$
\mathbb{E}(\Pi_{BDO}^H | B_1 = BLO, v = v^H) = \theta_2^I \left( v^H - \frac{A_1^1 + B_1^2 + \tau}{2} \right) = \theta_2^I \left( \kappa - \frac{1}{2} \right) \tau \geq 0, \text{ and}
$$

$$
\mathbb{E}(\Pi_{BLO}^H | B_1 = BLO, v = v^H) = 0,
$$

since the probability of execution of any LO at $t = 2$ is null. Hence, the trader will not choose a LO or NT since these options are strictly dominated by a MO.

### 2.2.4. $(A_1^1, B_1^2)$

Note that during the first trading period a trader has arrived at the market and has chosen a SMO (denoted by $B_1 = SMO$). The expected profits for the various types of orders at $t = 2$ are as follows:

$$
\mathbb{E}(\Pi_{BMO}^H | B_1 = SMO, v = v^H) = v^H - A_1^1 = (\kappa - k_1) \tau > 0,
$$

$$
\mathbb{E}(\Pi_{BDO}^H | B_1 = SMO, v = v^H) = \theta_2^I \left( v^H - \frac{A_1^1 + B_1^2}{2} \right) = \theta_2^I \left( \kappa + \frac{k_2 - k_1}{2} \right) \tau \geq 0, \text{ and}
$$

$$
\mathbb{E}(\Pi_{BLO}^H | B_1 = SMO, v = v^H) = 0,
$$

since the probability of execution of any LO at $t = 2$ is null. Hence, the trader will not choose a LO or NT because these options are strictly dominated by a MO.

### 2.2.5. $(A_1^1 - \tau, B_1^1)$

Note that during the first trading period a trader has arrived at the market and has chosen a SLO which improved the ask price (denoted by $B_1 = SLO$). The expected profits for the various types
of orders at \( t = 2 \) are as follows:

\[
\begin{align*}
\mathbb{E} \left( \Pi^H_{BMO,2} | B_1 = SLO, v = v^H \right) &= v^H - A^1 + \tau = (\kappa - k_1 + 1) \tau > 0, \\
\mathbb{E} \left( \Pi^H_{BDO,2} | B_1 = SLO, v = v^H \right) &= \theta^H_2 \left( v^H - A^1 - \tau + B^1_1 \frac{2}{2} \right) = \theta_2^H \left( \kappa + \frac{1}{2} \right) \tau \geq 0, \text{ and} \\
\mathbb{E} \left( \Pi^H_{BLO,2} | B_1 = SLO, v = v^H \right) &= 0,
\end{align*}
\]

since the probability of execution of any \( LO \) at \( t = 2 \) is null. Hence, the trader will not choose a \( LO \) or \( NT \) since these options are strictly dominated by a \( MO \).

2.3. State \( (t = 2, IL) \)

Note that the expected profits for an informed trader when \( v = v^L \) (IL) are similar to the expected profits of an \( IH \) trader when \( v = v^H \) since there exists the following symmetry. When the state of the \( LOB \) is \((A^1, B^1)\), an IL and an IH always make the same choice (apart from the direction of the order). When the state of the \( LOB \) is \((A^2, B^1)\), an IL chooses the same type of order as an IH when the state of the \( LOB \) is \((A^1, B^1 + \tau)\). When the state of the \( LOB \) is \((A^1, B^2)\), an IL chooses the same type of order as an IH when the state of the \( LOB \) is \((A^1 - \tau, B^1)\). When the state of the \( LOB \) is \((A^2, B^1)\), an IL chooses the same type of order as an IH when the state of the \( LOB \) is \((A^1 - \tau, B^1)\). When the state of the \( LOB \) is \((A^1, B^1 + \tau)\), an IL chooses the same type of order as an IH when the state of the \( LOB \) is \((A^1, B^1 + \tau)\).

2.4. State \( (t = 2, UB) \)

The best prices of the \( LOB \) at the beginning of the second trading period can take the following values: \((A^1, B^1), (A^1, B^2), (A^1, B^1 + \tau), (A^1, B^2), \) or \((A^1 - \tau, B^1)\).

2.4.1. \((A^1, B^1)\)

This occurs when either when a trader arriving at the market at \( t = 1 \) chooses to go to the \( DP \) or chooses \( NT \). Therefore, \( B_1 = \emptyset \).

Note that the possible cases such that the prices of the \( LOB \) have not changed during the first trading period are the following: 1) an informed trader who observes \( v^H \) and who goes to the dark at \( t = 1 \); 2) an informed trader who observes \( v^L \) and who goes to the dark at \( t = 1 \); 3) an uninformed buyer who goes to the dark at \( t = 1 \); 4) an uninformed seller who goes to the dark at \( t = 1 \); 5) an uninformed buyer who selects \( NT \) at \( t = 1 \); and 6) an uninformed seller who selects \( NT \) at \( t = 1 \). Hence,

\[
\mathbb{E}(\tilde{v} | B_1 = \emptyset) = \frac{\pi^2 \Omega_3 v^H + \frac{\pi^2}{2} \Omega_3 v^L + (1 - \pi)(\Gamma_0 + \Gamma_3) \mu}{\pi \Omega_3 + (1 - \pi)(\Gamma_0 + \Gamma_3)} = \mu,
\]

and because we focus on symmetric equilibria this state of the \( LOB \) does not gives any information to the uninformed trader. The expected profits for the various types of orders at \( t = 2 \) are as
follows:
\[
\begin{align*}
\mathbb{E}(\Pi_{BMO,2}^U|B_1 = \emptyset) &= \mathbb{E}(\bar{v}|B_1 = \emptyset) - A_1^1 = (\mu - \mu - k_1 \tau) = -k_1 \tau < 0, \text{ and} \\
\mathbb{E}(\Pi_{BDO,2}^U|B_1 = \emptyset) &= \theta_2^U \left( \mathbb{E}(\bar{v}|B_1 = \emptyset) - \frac{A_1^1 + B_1^1}{2} \right) = \theta_2^U \kappa (\mu - \mu) \tau = 0.
\end{align*}
\]

For the expected profits of a LO, note that a BLO chosen by an uninformed trader who arrives at the market at \( t = 2 \) can only be executed if at \( t = 1 \) there is a seller who chooses a DO and the order is not executed. Therefore, the probability of execution of a BLO chosen by an uninformed trader that faces \((A_1^1, B_1^1)\), which is denoted by \( p_{BLO,2}^U(B_1 = \emptyset) = p(B_2^{End} = SMO|B_1 = \emptyset) \), is given by
\[
\begin{align*}
p_{BLO,2}^U(B_1 = \emptyset) &= \frac{(1 - \theta_1^L) \frac{\pi}{2} \Omega_3 + (1 - \theta_1^L) \frac{1 - \pi}{2} \Gamma_3}{\pi \Omega_3 + (1 - \pi) (\Gamma_0 + \Gamma_3)},
\end{align*}
\]
and
\[
\begin{align*}
\mathbb{E}(\Pi_{BLO,2}^U|B_1 = \emptyset) &= \frac{p_{BLO,2}^U(B_1 = \emptyset)}{p_{BLO,2}^U(B_1 = \emptyset)} \delta(\mathbb{E}(\bar{v}|B_1 = \emptyset, B_2^{End} = SMO) - B_1^1 - \tau) \\
&= \frac{p_{BLO,2}^U(B_1 = \emptyset)}{p_{BLO,2}^U(B_1 = \emptyset)} \delta(k_1 - Z \kappa - 1) \tau,
\end{align*}
\]

since
\[
\begin{align*}
\mathbb{E}(\bar{v}|B_1 = \emptyset, B_2^{End} = SMO) &= \frac{(1 - \theta_1^L) (1 - \pi) \Gamma_3}{(1 - \theta_1^L) \pi \Omega_3 + (1 - \theta_1^L) (1 - \pi) \Gamma_3} \mu + \\
&\quad + \frac{(1 - \theta_1^L) \pi \Omega_3}{(1 - \theta_1^L) \pi \Omega_3 + (1 - \theta_1^L) (1 - \pi) \Gamma_3} v^L \\
&= \mu - Z \kappa \tau.
\end{align*}
\]

In this case the UB never chooses a MO.

\subsection{(A_1^1, B_1^1)}

This occurs when at \( t = 1 \) a trader arrives at the market and chooses a BMO (denoted by \( B_1 = BMO \)). In this case the LOB has changed in the ask side: \( A_1^2 = A_1^1 \) and \( B_1^2 = B_1^1 \). Note that
\[
\begin{align*}
\mathbb{E}(\bar{v}|B_1 = BMO) &= \mu + \frac{\lambda \pi \Omega_1}{1 - \lambda + \lambda \pi \Omega_1 + \lambda (1 - \pi) \Gamma_1} \kappa \tau = \mu + X \kappa \tau.
\end{align*}
\]

The expected profits for the various types of orders at \( t = 2 \) are as follows:
\[
\begin{align*}
\mathbb{E}(\Pi_{BMO,2}^U|B_1 = BMO) &= \mathbb{E}(\bar{v}|B_1 = BMO) - A_1^2 = (X \kappa - k_2) \tau, \\
\mathbb{E}(\Pi_{BDO,2}^U|B_1 = BMO) &= \theta_2^U \left( \mathbb{E}(\bar{v}|B_1 = BMO) - \frac{A_1^2 + B_1^1}{2} \right) = \theta_2^U \left( X \kappa - \frac{k_2 - k_1}{2} \right) \tau, \text{ and} \\
\mathbb{E}(\Pi_{BLO,2}^U|B_1 = BMO) &= 0,
\end{align*}
\]
since the probability of execution of any LO is null.
2.4.3. \((A_1^1, B_1^1 + \tau)\)

Note that the \(LOB\) has changed in the bid side because at \(t = 1\) a trader has arrived at the market and has chosen an improving \(BLO\) (denoted by \(B_1 = BLO\)). Note that

\[
E(\tilde{v}|B_1 = BLO) = \mu + \frac{\pi \Omega_2 \kappa}{\pi \Omega_2 + (1 - \pi) \Gamma_2} \tau = \mu + Y\kappa\tau.
\]

The expected profits for the various types of orders at \(t = 2\) are as follows:

\[
E(P_{BMO,2}^U|B_1 = BLO) = E(\tilde{v}|B_1 = BLO) - A_1^1 = (Y\kappa - k_1) \tau,
\]

\[
E(P_{BDO,2}^U|B_1 = BLO) = \frac{\theta_2^U}{2} \left( E(\tilde{v}|B_1 = BLO) - \frac{A_1^1 + B_1^1 + \tau}{2} \right) = \frac{\theta_2^U}{2} \left( Y\kappa - \frac{1}{2} \right) \tau, \quad \text{and}
\]

\[
E(P_{BLO,2}^U|B_1 = BLO) = 0.
\]

2.4.4. \((A_1^1, B_1^2)\)

This occurs when at \(t = 1\) a trader arrives at the market and chooses a \(SMO\) (i.e., \(B_1 = SMO\)). Note that

\[
E(\tilde{v}|B_1 = SMO) = \frac{1 - \lambda + \lambda (1 - \pi) \Gamma_1}{1 - \lambda + \lambda (1 - \pi) \Gamma_1 + \lambda \pi \Omega_1} \mu + \frac{\lambda \pi \Omega_1}{1 - \lambda + \lambda (1 - \pi) \Gamma_1 + \lambda \pi \Omega_1} \nu_L = \mu - X\kappa\tau.
\]

The expected profits for the various types of orders at \(t = 2\) are as follows:

\[
E(P_{BMO,2}^U|B_1 = SMO) = E(\tilde{v}|B_1 = SMO) - A_1^1 = -(X\kappa + k_1) \tau < 0,
\]

\[
E(P_{BDO,2}^U|B_1 = SMO) = \frac{\theta_2^U}{2} \left( E(\tilde{v}|B_1 = SMO) - \frac{A_1^1 + B_1^2}{2} \right) = -\frac{\theta_2^U}{2} \left( X\kappa - \frac{k_2 - k_1}{2} \right) \tau, \quad \text{and}
\]

\[
E(P_{BLO,2}^U|B_1 = SMO) = 0.
\]

2.4.5. \((A_1^1 - \tau, B_1^1)\)

Note that at \(t = 1\) a trader has arrived at the market and has chosen an improving \(SLO\) (i.e., \(B_1 = SLO\)). Note that

\[
E(\tilde{v}|B_1 = SLO) = \frac{(1 - \pi) \Gamma_2}{\pi \Omega_2 + (1 - \pi) \Gamma_2} \mu + \frac{\pi \Omega_2}{\pi \Omega_2 + (1 - \pi) \Gamma_2} \nu_L = \mu - Y\kappa\tau.
\]

The expected profits for the various types of orders at \(t = 2\) are as follows:

\[
E(P_{BMO,2}^U|B_1 = SLO) = E(\tilde{v}|B_1 = SLO) - A_1^1 + \tau = -(Y\kappa + k_1 - 1) \tau \leq 0,
\]

\[
E(P_{BDO,2}^U|B_1 = SLO) = \frac{\theta_2^U}{2} \left( E(\tilde{v}|B_1 = SLO) - \frac{A_1^1 - \tau + B_1^1}{2} \right) = -\frac{\theta_2^U}{2} \left( Y\kappa - \frac{1}{2} \right) \tau, \quad \text{and}
\]

\[
E(P_{BLO,2}^U|B_1 = SLO) = 0.
\]

In this case the \(UB\) chooses neither a \(MO\) nor a \(LO\).
2.5. State ($t = 2$, US)

Idem sub-section 2.3.

$t = 1$

There are 5 possible states depending on which type of trader arrived ($LT, IH, IL, UB, US$).

1.1. State ($t = 1$, Liquidity trader)

In this case a liquidity trader arrives. Its probability is $1 - \lambda$. He will choose a $BMO$ with probability $1/2$ and a $SMO$ with probability $1/2$.

1.2. State ($t = 1$, IH)

The $LOB$ starts from its original best prices ($A_1^1, B_1^1$). The expected profits for the various types of orders at $t = 1$ are as follows:

$$E(\Pi_{BMO,1}^{IH}|v = v^H) = v^H - A_1^1 = (\kappa - k_1)\tau > 0.$$  

For the expected profits of a $LO$, note that a $BLO$ chosen by an $IH$ who arrives at $t = 1$ is executed only if at $t = 2$ there is a trader who chooses a $SMO$. However, as a $US$ observing a $BLO$ will never choose a $MO$, we conclude that the probability of execution of the $LO$ is $(1 - \lambda)/2$. Hence,

$$E(\Pi_{BLO,1}^{IH}|v = v^H) = \frac{\delta (1 - \lambda)}{2} (v^H - B_1^1 - \tau) = \frac{\delta (1 - \lambda)}{2} (\kappa + k_1 - 1)\tau.$$  

For the expected profits of a $DO$, note that there are two cases: 1) the order is executed in the $DP$ and 2) the order is not executed in the $DP$ and, then, either it is cancelled or it returns at the exchange at the end of the second trading period as a $BMO$. In such a case, there are three possible ask prices, which depend on what has happened at $t = 2$:

1) $A_1^1 - \tau$: This ask price occurs if the trader at $t = 2$ chooses a $SLO$ (which can only be chosen by an uninformed seller).

2) $A_2^1$: This ask price occurs if the trader at $t = 2$ selects a $BMO$ (which can be chosen by an informed buyer or a liquidity trader). Notice, that an uninformed buyer observing no change in the $LOB$ will not choose a $MO$ (see 2.4.1.).

3) $A_1^1$: This ask price occurs all the other times.

---

1Note that we do not consider the case that the order returns as a $BLO$ because its probability of execution is zero.
Therefore,

\[
\mathbb{E}(\Pi_{BDO,1}^H|v = v^H) = \theta_1^H \left(v^H - A_1^H + B_1^H\right) + \left(1 - \theta_1^H\right) \max\left\{0, \delta \left[\lambda \left(1 - \frac{1}{2}\right) I_{SLO,2}^{\cup\cup, B_1 = \emptyset} (v^H - A_1^H + \tau) + \left(\lambda \pi I_{BMO,2}^{\cup\cup, B_1 = \emptyset} + \frac{1 - \lambda}{2}\right) (v^H - A_1^H) \right] \right\} = \theta_1^H \kappa \tau + \left(1 - \theta_1^H\right) \max\left\{0, \delta \left[\lambda \left(1 - \frac{1}{2}\right) I_{SLO,2}^{\cup\cup, B_1 = \emptyset} (k - k_1) + \right. \right. \\
+ \left. \left. \left(\lambda \pi I_{BMO,2}^{\cup\cup, B_1 = \emptyset} + \frac{1 - \lambda}{2}\right) (k - k_2) \right] \right\},
\]

where

\[
I_{SLO,2}^{\cup\cup, B_1 = \emptyset} = \begin{cases} 1, & \text{if at } t = 2, \text{ an } US \text{ selects a } SLO \text{ when } B_1 = \emptyset \\ 0, & \text{otherwise}, \end{cases}
\]

\[
= \begin{cases} 1, & \text{if } \frac{(1 - \theta_1^H)\frac{1}{2}\Omega_3 + (1 - \theta_1^H)\frac{1 - \pi}{2}\Gamma_3}{\pi \Omega_3 + (1 - \pi)(\Gamma_0 + \Gamma_3)} \delta (k_1 - 1 - Z_\kappa > 0), \\ 0, & \text{otherwise}, \end{cases}
\]

and

\[
I_{BMO,2}^{\cup\cup, B_1 = \emptyset} = \begin{cases} 1, & \text{if at } t = 2, \text{ an } IH \text{ selects a } BMO \text{ when } B_1 = \emptyset \\ 0, & \text{otherwise}, \end{cases}
\]

\[
= \begin{cases} 1, & \kappa - k_1 \geq \max\left\{\theta_2^H \kappa, \frac{(1 - \theta_1^H)\frac{1 - \pi}{2}\Gamma_3}{\pi \Omega_3 + (1 - \pi)(\Gamma_0 + \Gamma_3)} \delta (k_1 + \kappa - 1) \right\}, \\ 0, & \text{otherwise}. \end{cases}
\]

Simplifying, we obtain that:

\[
\mathbb{E}(\Pi_{BDO,1}^H|v = v^H) = \theta_1^H \kappa \tau + \left(1 - \theta_1^H\right) \max\left\{0, \delta \left[\lambda \left(1 - \frac{1}{2}\right) I_{SLO,2}^{\cup\cup, B_1 = \emptyset} + (k - k_1) \right] \right\} = \theta_1^H \kappa \tau + \left(1 - \theta_1^H\right) \delta \left[\lambda \left(1 - \frac{1}{2}\right) I_{SLO,2}^{\cup\cup, B_1 = \emptyset} + (k - k_1) \right] \tau,
\]

The last equality indicates that when an informed buyer selects to go to the DP at \( t = 1 \) and the order is not executed, it is optimal for him to choose a MO, which returns to the exchange at the end of the second trading period.
An informed trader never chooses NT as it is dominated at least by the MO.

1.3 State \((t = 1, \text{IL})\)

Due to the symmetry of the model, the expected profits of an IL trader are the same as the ones of an IH trader.

1.4. State \((t = 1, \text{UB})\)

The LOB starts from its original best prices \((A_1^1, B_1^1)\). The expected profits for the various types of orders at \(t = 1\) are as follows:

\[
E (\Pi_{BMO,1}^{UB}) = \mu - A_1^1 = -k_1 \tau < 0.
\]

For the expected profits of a LO, note that the order gets executed if the next order is SMO which can come from an informed trader at \(t = 2\) that chooses a SMO or from a liquidity trader. Note that an uninformed trader upon observing a BLO in \(t = 1\) will never choose a SMO. Thus,

\[
E (\Pi_{BLO,1}^{UB}) = p(B_2 = SMO|B_1 = BLO) E (\Pi_{BLO,1}^{UB}|B_1 = BLO, B_2 = SMO),
\]

where

\[
p(B_2 = SMO) = \frac{\lambda}{2} \pi_{IL, B_1 = BLO} + \frac{1 - \lambda}{2}
\]

and

\[
E (\Pi_{BLO,1}^{UB}|B_1 = BLO, B_2 = SMO) = \delta (E(\tilde{v}|B_1 = BLO, B_2 = SMO) - B_1^1 - \tau).
\]

Moreover, notice that

\[
E (\tilde{v}|B_1 = BLO, B_2 = SMO) = \mu - \frac{\lambda \pi_{IL, B_1 = BLO}}{1 - \lambda + \lambda \pi_{IL, B_1 = BLO}^\kappa} \tau.
\]

So,

\[
E (\Pi_{BLO,1}^{UB}|B_1 = BLO, B_2 = SMO) = \delta \left( k_1 - 1 - \frac{\lambda \pi_{IL, B_1 = BLO}}{1 - \lambda + \lambda \pi_{IL, B_1 = BLO}^\kappa} \right) \tau,
\]

and

\[
E (\Pi_{BLO,1}^{UB}) = \delta \left( \frac{\lambda}{2} \pi_{IL, B_1 = BLO} + \frac{1 - \lambda}{2} \right) \delta \left( k_1 - 1 - \frac{\lambda \pi_{IL, B_1 = BLO}}{1 - \lambda + \lambda \pi_{IL, B_1 = BLO}^\kappa} \right) \tau
\]

\[
= \delta \left( (1 - \lambda) (k_1 - 1) - \lambda \pi_{IL, B_1 = BLO}^\kappa (k_1 + 1) \right) \tau.
\]
with

\[
I_{IL,B_1=BLO}^{IL,B_1=BLO} = \begin{cases} 
1, & \text{if at } t = 2, \text{ an } IL \text{ selects a } SMO \text{ when } B_1 = BLO, \\
0, & \text{otherwise,}
\end{cases}
\]

\[
= \begin{cases} 
1, & \text{if } \theta_2^I \leq \frac{\kappa - k_1 + 1}{\kappa + 1}, \\
0, & \text{otherwise.}
\end{cases}
\]

For the expected profits if the trader chooses to go to the DP, then there are two cases: 1) the order is executed in the DP and 2) the order is not executed in the DP and, then, either it is cancelled or it returns at the exchange at the end of the second trading period as a BMO. In such a case, there are three possible ask prices, which depend on what has happened at \( t = 2 \).

1) \( A_1^1 - \tau \): This ask price occurs if the trader at \( t = 2 \) chooses a SLO (which can be chosen by either an informed seller or an uninformed seller).

2) \( A_2^1 \): This ask price occurs if the trader at \( t = 2 \) decides a BMO (which can be chosen either by an informed buyer or a liquidity trader). Notice, that an uninformed buyer observing no change in the LOB will not choose a MO (see 2.4.1).

3) \( A_3^1 \): This ask price occurs all the other times.

\[
E(\Pi_{UB,BDO,1}^{UB,BDO,1}) = \theta_U^I \left( \mu - \frac{A_1^1 + B_1^1}{2} \right) + \\
+ \max \left\{ 0, (1 - \theta_U^I) \delta \left[ \beta (1 - \pi I_{IL,B_1=\varnothing}^{IL,B_1=\varnothing} + (1 - \pi) I_{US,B_1=\varnothing}^{US,B_1=\varnothing}) (E(\tilde{v}|B_1 = \varnothing, B_2 = SLO) - A_1^1 + \tau) + \\
+ \left( \frac{\lambda}{2} I_{IH,B_1=\varnothing}^{IH,B_1=\varnothing} + 1 - \frac{\lambda}{2} \right) (E(\tilde{v}|B_1 = \varnothing, B_2 = BMO) - A_1^1) + \\
+ \left( 1 - \frac{\lambda}{2} I_{SLO,2}^{IL,B_1=\varnothing} + (1 - \pi) I_{SLO,2}^{US,B_1=\varnothing} \right) - \left( \frac{\lambda}{2} I_{BMO,2}^{IH,B_1=\varnothing} + 1 - \frac{\lambda}{2} \right) \right] \times \\
\times \left( E(\tilde{v}|B_1 = \varnothing, B_2 \neq SLO, BMO) - A_1^1 \right) \right\},
\]

where

\[
I_{IL,B_1=\varnothing}^{IL,B_1=\varnothing} = \begin{cases} 
1, & \text{if at } t = 2, \text{ an } IL \text{ selects a } SLO \text{ when } B_1 = \varnothing, \\
0, & \text{otherwise,}
\end{cases}
\]

\[
I_{US,B_1=\varnothing}^{US,B_1=\varnothing} = \begin{cases} 
1, & \text{if at } t = 2, \text{ an } US \text{ selects a } SLO \text{ when } B_1 = \varnothing, \\
0, & \text{otherwise,}
\end{cases}
\]

\[
I_{IH,B_1=\varnothing}^{IH,B_1=\varnothing} = \begin{cases} 
1, & \text{if at } t = 2, \text{ an } IH \text{ selects a } BMO \text{ when } B_1 = \varnothing, \\
0, & \text{otherwise.}
\end{cases}
\]
After some computations, it follows that

\[ I_{UB,1} = \begin{cases} 1, & \text{if } \frac{1-\pi}{2} (1 - \theta_U^1 \Gamma_3 \delta(k_1 + \kappa - 1) > \kappa - k_1, \\
0, & \text{otherwise,} \end{cases} \]

and

\[ I_{UB,2} = \begin{cases} 1, & \text{if } \frac{1-\pi}{2} (1 - \theta_U^1 \Gamma_3 \delta(k_1 + \kappa - 1) \geq \theta_2 \kappa, \\
0, & \text{otherwise,} \end{cases} \]

Note that

\[ E(\bar{v} | B_1 = \emptyset, B_2 = SLO) = \mu - \frac{\pi I_{IL,B_1 = \emptyset}^{IL,SLO,2}}{\pi I_{IL,B_1 = \emptyset}^{IL,SLO,2} + (1 - \pi) I_{US,B_1 = \emptyset}^{US,SLO,2}} \kappa \tau, \]

\[ E(\bar{v} | B_1 = \emptyset, B_2 = BMO) = \mu + \frac{\lambda \pi I_{IH,B_1 = \emptyset}^{IH,BMO,2}}{1 - \lambda + \lambda \pi I_{IH,B_1 = \emptyset}^{IH,BMO,2}} \kappa \tau, \text{ and} \]

\[ E(\bar{v} | B_1 = \emptyset, B_2 \neq SLO, BMO) = \mu + \frac{\lambda \pi I_{IL,O_1 = \emptyset}^{IL,SLO,2} - (1 - \lambda) I_{US,O_1 = \emptyset}^{US,SLO,2} - (1 - \lambda) I_{IB,O_1 = \emptyset}^{IB,SLO,2}}{1 - \lambda + \lambda \pi I_{IH,B_1 = \emptyset}^{IH,BMO,2}} \kappa \tau. \]

Therefore,

\[ E(\Pi_{UB,1}^{\emptyset}) = \max \left\{ \mu - \frac{\pi I_{IL,B_1 = \emptyset}^{IL,SLO,2}}{\pi I_{IL,B_1 = \emptyset}^{IL,SLO,2} + (1 - \pi) I_{US,B_1 = \emptyset}^{US,SLO,2}} \kappa \tau - \mu - k_1 \tau + \tau \right\} + \left\{ \frac{1 - \lambda}{2} + \frac{\lambda}{2} I_{IH,B_1 = \emptyset}^{IH,BMO,2} \left( \mu + \frac{\lambda \pi I_{IH,B_1 = \emptyset}^{IH,BMO,2}}{1 - \lambda + \lambda \pi I_{IH,B_1 = \emptyset}^{IH,BMO,2}} \kappa \tau - \mu - k_2 \tau \right) + \left\{ \frac{1 - \lambda}{2} I_{IL,B_1 = \emptyset}^{IL,SLO,2} + (1 - \pi) I_{US,B_1 = \emptyset}^{US,SLO,2} - (1 - \lambda) I_{IB,B_1 = \emptyset}^{IB,SLO,2} \right\} \right\} \times \frac{\lambda \pi I_{IH,B_1 = \emptyset}^{IH,BMO,2} I_{IL,B_1 = \emptyset}^{IL,SLO,2} - I_{IL,B_1 = \emptyset}^{IL,SLO,2} \kappa \tau - k_1 \tau \right\} \]
or, equivalently,

\[ \mathbb{E}(\Pi_{BDO,1}^{UB}) = \max \left\{ 0, (1 - \theta_1^U) \delta \left[ \frac{\lambda}{2} \left( \pi I_{SLO,2}^{IL,B_1=\emptyset} + (1 - \pi) I_{SLO,2}^{US,B_1=\emptyset} \right) - \frac{k_2}{2} \left( 1 - \lambda + \lambda \pi I_{BMO,2}^{IH,B_1=\emptyset} \right) k_1 \left( \frac{1 - \lambda}{2} + \frac{\lambda}{2} I_{BMO,2}^{IH,B_1=\emptyset} - 1 \right) \right] \right\} \]

\[ = \max \left\{ 0, (1 - \theta_1^U) \delta \left[ \frac{\lambda}{2} \left( \pi I_{SLO,2}^{IL,B_1=\emptyset} + (1 - \pi) I_{SLO,2}^{US,B_1=\emptyset} \right) + \frac{1 - \lambda}{2} \pi I_{BMO,2}^{IH,B_1=\emptyset} k_1 - k_2 \right] \right\} = 0. \]

The last equality indicates that when an uninformed buyer selects to go to the DP at \( t = 1 \) and the order is not executed, it is optimal for him to choose to cancel the order. To understand why the payoff at \( t = 1 \) of a \( BDO - BMO \) for the uninformed trader is always negative, let us rewrite the corresponding payoff as

\[ \theta_1^U \cdot 0 + (1 - \theta_1^U) \delta \left( -k_1 \tau + \frac{\lambda}{2} \left( \pi I_{SLO,2}^{IL,B_1=\emptyset} + (1 - \pi) I_{SLO,2}^{US,B_1=\emptyset} \right) + \left( \frac{\lambda}{2} I_{BMO,2}^{IH,B_1=\emptyset} + \frac{1 - \lambda}{2} \right) (k_2 - k_1) \right). \]

This is because if a \( BDO \) is executed at \( t = 1 \), then its expected profits are zero, which occurs with probability \( \theta_1^U \). If the order is not executed at \( t = 1 \) and returns to the market at the end of the second trading period, which occurs with probability \( 1 - \theta_1^U \), then expected profits depend on whether the uninformed trader who returns to the exchange decides to submit a \( NT \) or MO. If the uninformed trader selects \( NT \) then the expected profits equal zero. If he submits a \( MO \) then the profit consists of three terms. The first consists of the expected profits of a \( BMO \) at \( t = 1 \) for an \( UB \) (i.e., \(-k_1 \tau \)). The second is the potential increase in profits due to the possibility that at \( t = 2 \) a new trader arrives and submits a \( SLO \) leading to a better price for the uninformed buyer (given by \( \frac{\lambda}{2} \left( \pi I_{SLO,2}^{IL,B_1=\emptyset} + (1 - \pi) I_{SLO,2}^{US,B_1=\emptyset} \right) \tau \)). The third is the potential decrease in profits due to the possibility that a trader at \( t = 2 \) submits a \( BMO \) and, consequently, the \( MO \) that arrives at the end of the second trading period from the \( DP \) is executed at a worse price (given by \( - \left( \frac{\lambda}{2} I_{BMO,2}^{IH,B_1=\emptyset} + \frac{1 - \lambda}{2} \right) (k_2 - k_1) \tau \)). We find that the increase in expected profits due to the potential arrival of a \( SLO \) at \( t = 2 \) is not greater than \(-k_1 \tau \) and these losses might be even greater in case that a \( BMO \) is submitted at \( t = 2 \). Consequently, the payoff at \( t = 1 \) of the \( BDO - BMO \) for the uninformed buyer is always negative.

It is important to point out that the expected profits of a \( DO \) are negative. Therefore, we conclude that an \( UB \) never goes to the \( DP \) at \( t = 1 \).

1.5. State (\( t = 1, \text{US} \))

Note that the game is symmetric and, therefore, the uninformed seller and the uniformed buyer have identical expected profits. The expected profits for the various types of orders at \( t = 1 \) are as follows:

\[ \mathbb{E}(\Pi_{SMO,1}^{US}) = -k_1 \tau < 0, \]
\[
\begin{align*}
\mathbb{E}(\Pi_{SLO,1}^{US}) &= \frac{\delta}{2} \left[ (1 - \lambda + \lambda \pi \theta |_{BMO,2}^{IH,B_2=SLO}) (k_1 - 1) - \lambda \pi |_{BMO,2}^{IH,B_2=SLO} \right] \tau, \text{ and} \\
\mathbb{E}(\Pi_{SDO,1}^{US}) &= \max \left\{ 0, (1 - \theta_U^1) \delta \left[ \frac{\lambda}{2} \left( \pi |_{BLO,2}^{IH,B_1=\emptyset} + (1 - \pi) |_{BLO,2}^{UB,B_1=\emptyset} \right) + \left( 1 - \lambda + \frac{\lambda \pi}{2} |_{SMO,2}^{IH,B_1=\emptyset} \right) (k_1 - k_2) - k_1 \right] \tau \right\} = 0,
\end{align*}
\]

with
\[
\begin{align*}
|_{BLO,2}^{IH,B_1=\emptyset} &= \begin{cases} 
1, & \text{if at } t = 2, \text{ an IH selects a BLO when } B_1 = \emptyset, \\
0, & \text{otherwise},
\end{cases} \\
|_{BLO,2}^{UB,B_1=\emptyset} &= \begin{cases} 
1, & \text{if at } t = 2, \text{ an UB selects a BLO when } B_1 = \emptyset, \\
0, & \text{otherwise},
\end{cases} \\
|_{SMO,2}^{IH,B_1=\emptyset} &= \begin{cases} 
1, & \text{if at } t = 2, \text{ an IL selects a SMO when } B_1 = \emptyset, \\
0, & \text{otherwise}.
\end{cases}
\]

**Summary**

To summarize we can tabulate the expected profits for an informed buyer and seller at \( t = 2 \) are summarized in Table I.1 and Table I.2, respectively.

<table>
<thead>
<tr>
<th>IH</th>
<th>BMO</th>
<th>BDO</th>
<th>BLO</th>
<th>NT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1^1, B_1^1 )</td>
<td>( (k - k_1) \tau )</td>
<td>( \theta_1^1 \kappa \tau )</td>
<td>( P_1 \delta (k_1 + \kappa - 1) \tau )</td>
<td>0</td>
</tr>
<tr>
<td>( A_2^1, B_1^1 )</td>
<td>( (k - k_2) \tau )</td>
<td>( \theta_2^1 \left( \kappa - \frac{k_2 - k_1}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( A_1^1, B_1^1 + \tau )</td>
<td>( (k - k_1) \tau )</td>
<td>( \theta_2^1 \left( \kappa + \frac{1}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( A_1^1, B_2^1 )</td>
<td>( (k - k_1) \tau )</td>
<td>( \theta_2^1 \left( \kappa + \frac{k_2 - k_1}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( A_1^1 - \tau, B_1^1 )</td>
<td>( (k - k_1 + 1) \tau )</td>
<td>( \theta_2^1 \left( \kappa + \frac{1}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table I.1: Expected profits of an informed buyer (IH) at \( t = 2 \)

<table>
<thead>
<tr>
<th>IL</th>
<th>SMO</th>
<th>SDO</th>
<th>SLO</th>
<th>NT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1^1, B_1^1 )</td>
<td>( (k - k_1) \tau )</td>
<td>( \theta_2^1 \kappa \tau )</td>
<td>( P_1 \delta (k_1 + \kappa - 1) \tau )</td>
<td>0</td>
</tr>
<tr>
<td>( A_2^1, B_1^1 )</td>
<td>( (k - k_1) \tau )</td>
<td>( \theta_2^1 \left( \kappa - \frac{k_1 - k_2}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( A_1^1, B_1^1 + \tau )</td>
<td>( (k - k_1 + 1) \tau )</td>
<td>( \theta_2^1 \left( \kappa + \frac{1}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( A_1^1, B_2^1 )</td>
<td>( (k - k_2) \tau )</td>
<td>( \theta_2^1 \left( \kappa + \frac{k_1 - k_2}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( A_1^1 - \tau, B_1^1 )</td>
<td>( (k - k_1) \tau )</td>
<td>( \theta_2^1 \left( \kappa - \frac{1}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table I.2: Expected profits of an informed seller (IL) at \( t = 2 \)
where $P_I$ is the probability of execution of a LO placed by an informed trader at $t = 2$, and equals

$$P_I = p_{BLO}^{IH} (B_1 = \emptyset) = p_{SLO}^{IH} (B_1 = \emptyset) = \frac{(1 - \theta_I^U) \frac{1 - \pi}{2} \Gamma_3 - \pi \Omega_3}{\pi \Omega_3 + (1 - \pi)(\Gamma_0 + \Gamma_3)}.$$ 

Similarly, the expected profits of the uninformed buyer and seller at $t = 2$ are summarized in Table I.3 and Table I.4, respectively.

<table>
<thead>
<tr>
<th>UB</th>
<th>BMO</th>
<th>BDO</th>
<th>BLO</th>
<th>NT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(A_1^1, B_1^1)$</td>
<td>$-k_1 \tau$</td>
<td>$0$</td>
<td>$P_U \delta (k_1 - Z \kappa - 1) \tau$</td>
<td>$0$</td>
</tr>
<tr>
<td>$(A_1^2, B_1^1)$</td>
<td>$(X \kappa - k_2) \tau$</td>
<td>$\theta_2^U (X \kappa - k_2 - 1) \tau$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$(A_1^1, B_1^1 + \tau)$</td>
<td>$(Y \kappa - k_1) \tau$</td>
<td>$\theta_2^U (Y \kappa - \frac{k_1}{2}) \tau$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$(A_1^1, B_1^2)$</td>
<td>$-(X \kappa + k_1) \tau$</td>
<td>$-\theta_2^U (X \kappa - k_2 - 1) \tau$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$(A_1^1 - \tau, B_1^1)$</td>
<td>$-(Y \kappa + k_1 - 1) \tau$</td>
<td>$-\theta_2^U (Y \kappa - \frac{k_1}{2}) \tau$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Table I.3: Expected profits of an uninformed buyer (UB) at $t = 2$

<table>
<thead>
<tr>
<th>US</th>
<th>SMO</th>
<th>SDO</th>
<th>SLO</th>
<th>NT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(A_1^1, B_1^1)$</td>
<td>$-k_1 \tau$</td>
<td>$0$</td>
<td>$P_U \delta (k_1 - Z \kappa - 1) \tau$</td>
<td>$0$</td>
</tr>
<tr>
<td>$(A_1^2, B_1^1)$</td>
<td>$-(k_1 + X \kappa) \tau$</td>
<td>$-\theta_2^U (X \kappa - k_2 - 1) \tau$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$(A_1^1, B_1^1 + \tau)$</td>
<td>$-(k_1 + 1 + Y \kappa) \tau$</td>
<td>$-\theta_2^U (Y \kappa - \frac{k_1}{2}) \tau$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$(A_1^1, B_1^2)$</td>
<td>$(X \kappa - k_2) \tau$</td>
<td>$\theta_2^U (Y \kappa - k_2 - 1) \tau$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$(A_1^1 - \tau, B_1^1)$</td>
<td>$(Y \kappa - k_1) \tau$</td>
<td>$\theta_2^U (Y \kappa - \frac{k_1}{2}) \tau$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Table I.4: Expected profits of an uninformed seller (US) at $t = 2$

where $P_U$ is the probability of execution of a LO placed by an uninformed trader at $t = 2$, and equals

$$P_U = p_{BLO}^{UB} (B_1 = \emptyset) = p_{SLO}^{US} (B_1 = \emptyset) = \frac{(1 - \theta_U^I) \frac{1 - \pi}{2} \Omega_3 + \pi \Omega_3}{\pi \Omega_3 + (1 - \pi)(\Gamma_0 + \Gamma_3)}.$$ 

At $t = 1$ the expected profits of an informed $IH$ and an uniformed buyer $UB$ are summarized in Table I.5 and Table I.6, respectively. Notice that due to the symmetry of the game, the expected profits of an $IL$ trader and an $US$ are similar to the expected profits of an $IH$ and an $UB$ trader, respectively.
### Table I.5: Expected profits of an informed buyer (IH) at \( t = 1 \)

<table>
<thead>
<tr>
<th>IH</th>
<th>Expected profits at ( t = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( BMO )</td>
<td>((\kappa - k_1) \tau)</td>
</tr>
<tr>
<td>( BLO )</td>
<td>(\frac{\delta (1 - \lambda)}{2} (\kappa + k_1 - 1) \tau)</td>
</tr>
<tr>
<td>( BDO )</td>
<td>(\theta_1^I \kappa \tau + (1 - \theta_1^I) \delta \left(\lambda \frac{(1-\pi)(\lambda I_{SLO,1}^{IL,B_1=SLO,2})}{2} + (\kappa - k_1) - (k_2 - k_1) \left(\lambda \pi I_{BMO,2}^{I,B_1=SLO,2} + \frac{1-\lambda}{2}\right)\right) \tau)</td>
</tr>
<tr>
<td>( NT )</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table I.6: Expected profits of an uninformed buyer (UB) at \( t = 1 \)

<table>
<thead>
<tr>
<th>UB</th>
<th>Expected Profits at ( t = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( BMO )</td>
<td>(-k_1 \tau)</td>
</tr>
<tr>
<td>( BLO )</td>
<td>(\frac{\delta}{2} \left( (1 - \lambda) (k_1 - 1) - \lambda \pi I_{SMO,2}^{IL,B_1=BLO} (\kappa - k_1 + 1) \right) \tau)</td>
</tr>
<tr>
<td>( BDO )</td>
<td>0</td>
</tr>
<tr>
<td>( NT )</td>
<td>0</td>
</tr>
</tbody>
</table>

Table I.6: Expected profits of an uninformed buyer (UB) at \( t = 1 \)
Internet Appendix II (Step by step proofs of Proposition 1, Lemma C.2 and Proposition 2)

In this Appendix we completely characterize the equilibria in the single-venue market model (Proposition 1) and in the two-venue market model (Lemma C.2 and Proposition 2). For notations see the paper and Internet Appendix I.

Proof of Proposition 1. The procedure we follow to check if a particular strategy profile constitutes a PBE is as follows:

1. Specify a strategy profile for rational traders at $t = 1$.
2. Update the beliefs of the uninformed trader at $t = 2$ using Bayes’ rule at all information sets, whenever possible.
3. Given their beliefs, find the optimal response for the traders at $t = 2$.
4. Given the optimal response of traders at $t = 2$, and using Tables B.4 and B.5 in Appendix B in the paper find the optimal action for rational traders at $t = 1$.
5. Check if the optimal strategy profile for the traders at $t = 1$ coincide with the profile suggested in step 1.

We apply the procedure outlined above to check when each possible strategy profile can be an equilibrium.

Case A. Suppose that $k_1 > 1$.

$\mathcal{E}^{ND}_1$: $(BMO, SMO, BLO, SLO)$

First step. In this case $\Omega_0 = 0$, $\Omega_1 = 1$, $\Omega_2 = 0$, $\Gamma_0 = 0$, $\Gamma_1 = 0$, and $\Gamma_2 = 1$.

Second step. Using Bayes’ rule we obtain that $X^{1,ND} = \frac{\lambda \pi}{1 - \lambda + \lambda \pi}$ and $Y^{1,ND} = 0$.

Third step. Applying Lemma 1, we know that at $t = 2$ the optimal strategy of informed traders is to choose a $MO$, while the optimal strategy of the uninformed trader is as follows:
State of the book | UB | US |
--- | --- | --- |
$(A_1^1, B_1^1)$ | $NT$ | $NT$ |
$(A_2^1, B_1^1)$ | \(MO\) if \(\frac{\lambda \pi}{1 - \lambda + \lambda \pi} \kappa > k_2\) \(NT\) if \(\frac{\lambda \pi}{1 - \lambda + \lambda \pi} \kappa \leq k_2\) | $NT$ |
$(A_1^1, B_1^1 + \tau)$ | $NT$ | $NT$ |
$(A_1^1, B_2^1)$ | $NT$ | \(MO\) if \(\frac{\lambda \pi}{1 - \lambda + \lambda \pi} \kappa > k_2\) \(NT\) if \(\frac{\lambda \pi}{1 - \lambda + \lambda \pi} \kappa \leq k_2\) |
$(A_1^1 - \tau, B_1^1)$ | $NT$ | $NT$ |

Table II.1: Optimal responses of uninformed traders at \(t = 2\) when the strategy profile at \(t = 1\) is \((BMO, SMO, BLO, SLO)\).

**Fourth step.** Given the optimal response of traders at \(t = 2\), we find the optimal action for all rational traders at \(t = 1\).

Informed traders at \(t = 1\) have no incentives to deviate from the prescribed strategy profile whenever
\[
k - k_1 \geq \delta \frac{1 - \lambda}{2} (\kappa + k_1 - 1). \tag{II.1}
\]

Uninformed traders at \(t = 1\) have no incentives incentive to deviate from the prescribed strategy if and only if
\[
(1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) > 0. \tag{II.2}
\]

**Fifth step.** Nobody at \(t = 1\) has unilateral incentives to deviate from \((BMO, SMO, BLO, SLO)\) when both conditions \((II.1)\) and \((II.2)\) are satisfied, and these conditions can be rewritten as
\[
\sigma \geq \kappa^{I}_L - \frac{\lambda \pi}{\sigma - (k_1 - 1) \tau}, \tag{II.3}
\]
where
\[
\kappa^{I}_L = \frac{\delta (k_1 - 1) (1 - \lambda) + 2k_1}{2 - \delta (1 - \lambda)}, \quad PIN = \lambda \pi \quad \text{and} \quad \psi^{U}_L - NT = \frac{(1 - \lambda) (k_1 - 1) \tau}{\sigma - (k_1 - 1) \tau}.
\]

Finally, we consider the moves that are in the equilibrium path and must take into account that \((BMO, SMO, BLO, SLO)\) is the strategy profile chosen at \(t = 1\). Combining Expression \((II.2)\) and Table II.1 it follows that an uninformed traders always chooses \(NT\) at \(t = 2\).

\[E_4^{ND}: (BMO, SMO, NT, NT)\]

**First step.** In this case \(\Omega_0 = 0, \Omega_1 = 1, \Omega_2 = 0, \Gamma_0 = 1, \Gamma_1 = 0, \text{ and } \Gamma_2 = 0.\)

**Second step.** Using Bayes’ rule we obtain that \(X^{2,ND} = \frac{\lambda \pi}{1 - \lambda + \lambda \pi}\) and \(Y^{2,ND}\) is undetermined \(Y^{2,ND} \in [0, 1]\) (as Bayes’ rule implies \(Y^{2,ND} = \frac{0}{0}\)).

**Third step.** Applying Lemma 1, we know that at \(t = 2\) the optimal strategy of informed traders
is to choose a MO, while the optimal strategy of uninformed trader is:

\[
\begin{array}{|c|c|c|}
\hline
\text{State of the book} & \text{UB} & \text{US} \\
\hline
(A_1^1, B_1^1) & NT & NT \\
\hline
(A_2^2, B_1^1) & \begin{cases} MO \text{ if } \frac{\lambda \pi}{1 - \lambda + \lambda \pi} \kappa > k_2 \\ NT \text{ if } \frac{\lambda \pi}{1 - \lambda + \lambda \pi} \kappa \leq k_2 \end{cases} & NT \\
\hline
(A_1^1, B_1^1 + \tau) & \begin{cases} MO \text{ if } Y \kappa > k_1 \\ NT \text{ if } Y \kappa \leq k_1 \end{cases} & NT \\
\hline
(A_1^1, B_2^2) & NT & \begin{cases} MO \text{ if } \frac{\lambda \pi}{1 - \lambda + \lambda \pi} \kappa > k_2 \\ NT \text{ if } \frac{\lambda \pi}{1 - \lambda + \lambda \pi} \kappa \leq k_2 \end{cases} \\
\hline
(A_1^1 - \tau, B_1^1) & NT & \begin{cases} MO \text{ if } Y \kappa > k_1 \\ NT \text{ if } Y \kappa \leq k_1 \end{cases} \\
\hline
\end{array}
\]

Table II.2: Optimal responses of uninformed traders at \( t = 2 \) when the strategy profile at \( t = 1 \) is \((BMO, SMO, NT, NT)\).

**Fourth step.** Given the optimal response of traders at \( t = 2 \), we find the optimal action of rational traders at \( t = 1 \).

Informed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever

\[
\kappa - k_1 \geq \delta \frac{1 - \lambda}{2} (\kappa + k_1 - 1).
\] (II.4)

Uninformed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy if and only if

\[
0 \geq (1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)).
\] (II.5)

**Fifth step.** No trader at \( t = 1 \) has unilateral incentives to deviate from \((BMO, SMO, NT, NT)\) if and only if both conditions (II.4) and (II.5) are satisfied. Notice that these conditions can be rewritten as

\[
\sigma \geq \kappa_{MO-LO}^I \text{ and } PIN \geq \psi_{LO-NT}^U.
\]

Finally, in the following table we include the moves that are in the equilibrium path at \( t = 2 \) for an uninformed trader, taking into account the conditions that must be satisfied if the strategy profile chosen at \( t = 1 \) is \((BMO, SMO, NT, NT)\).
Table II.3: Optimal choice of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BMO, SMO, NT, NT)$.

$E_{3}^{ND}: (BLO, SLO, BLO, BLO)$

**First step.** In this case $\Omega_0 = 0$, $\Omega_1 = 0$, $\Omega_2 = 1$, $\Gamma_0 = 0$, $\Gamma_1 = 0$, and $\Gamma_2 = 1$.

**Second step.** Using Bayes’ rule we obtain that $X_{3}^{3,ND} = 0$ and $Y_{3,ND}^{3} = \pi$.

**Third step.** Applying Lemma 1, we know that at $t = 2$ the optimal strategy for informed traders is to choose a MO, while for the uninformed trader is:

<table>
<thead>
<tr>
<th>State of the book</th>
<th>UB</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(A_1^0, B_1^0)$</td>
<td>$NT$</td>
<td>$NT$</td>
</tr>
<tr>
<td>$(A_1^1, B_1^1)$</td>
<td>$NT$ if $\frac{\lambda \pi}{1 - \lambda + \lambda \pi} \kappa &gt; k_2$</td>
<td>$NT$ if $\frac{\lambda \pi}{1 - \lambda + \lambda \pi} \kappa \leq k_2$</td>
</tr>
<tr>
<td>$(A_0^1, B_1^1)$</td>
<td>$MO$ if $\frac{\lambda \pi}{1 - \lambda + \lambda \pi} \kappa &gt; k_2$</td>
<td>$NT$ if $\frac{\lambda \pi}{1 - \lambda + \lambda \pi} \kappa \leq k_2$</td>
</tr>
</tbody>
</table>

Table II.4: Optimal responses of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BLO, SLO, BLO, BLO)$.

**Fourth step.** Given the optimal response of traders at $t = 2$, we find the optimal action for the rational traders at $t = 1$.

Informed traders have no incentives to deviate at $t = 1$ from the prescribed strategy profile whenever

$$\delta \frac{1 - \lambda}{2} (\kappa + k_1 - 1) > \kappa - k_1. \quad (II.6)$$

Uninformed traders have no incentives to deviate at $t = 1$ from the prescribed strategy if and only if

$$(1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) > 0. \quad (II.7)$$

**Fifth step.** No trader at $t = 1$ has unilateral incentives to deviate from $(BLO, SLO, BLO, SLO)$ if and only if both conditions $(II.6)$ and $(II.7)$ are satisfied. Notice that these conditions can be
rewritten as
\[ \sigma < \kappa_{MO-LO}^I \text{ and } PIN < \psi_{LO-NT}^U. \]

Finally, in the following table we include the moves that are in the equilibrium path at \( t = 2 \) for an uninformed trader, taking into account the conditions that must be satisfied if \((BLO,SLO,BLO,SLO)\) is the strategy profile chosen at \( t = 1 \).

<table>
<thead>
<tr>
<th>State of the book</th>
<th>UB</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>((A_1^2,B_1^1))</td>
<td>NT</td>
<td>NT</td>
</tr>
<tr>
<td>((A_1^1,B_1^1 + \tau))</td>
<td>{ MO if ( \pi_\kappa \geq \kappa_1 ) }\ NT if ( \pi_\kappa \leq \kappa_1 )</td>
<td>NT</td>
</tr>
<tr>
<td>((A_1^1,B_1^1))</td>
<td>NT</td>
<td>NT</td>
</tr>
<tr>
<td>((A_1^1 - \tau,B_1^1))</td>
<td>NT</td>
<td>{ MO if ( \pi_\kappa &gt; \kappa_1 ) }\ NT if ( \pi_\kappa \leq \kappa_1 )</td>
</tr>
</tbody>
</table>

Table II.5: Optimal choice of uninformed traders at \( t = 2 \) when the strategy profile at \( t = 1 \) is \((BLO,SLO,BLO,SLO)\).

\( \mathcal{E}^{ND}_4 \): \((BLO,SLO,NT,NT)\)

**First step.** In this case \( \Omega_0 = 0, \Omega_1 = 0, \Omega_2 = 1, \Gamma_0 = 1, \Gamma_1 = 0, \) and \( \Gamma_2 = 0 \).

**Second step.** Using Bayes’ rule we obtain that \( X^{4,ND} = 0 \) and \( Y^{4,ND} = 1 \).

**Third step.** Applying Lemma 1, we know that at \( t = 2 \) the optimal strategy for informed traders is to choose a \( MO \), while for the uninformed trader is:

<table>
<thead>
<tr>
<th>State of the book</th>
<th>UB</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>((A_1^1,B_1^1))</td>
<td>NT</td>
<td>NT</td>
</tr>
<tr>
<td>((A_1^1,B_1^1))</td>
<td>NT</td>
<td>NT</td>
</tr>
<tr>
<td>((A_1^1,B_1^1 + \tau))</td>
<td>MO</td>
<td>NT</td>
</tr>
<tr>
<td>((A_1^1,B_1^1))</td>
<td>NT</td>
<td>NT</td>
</tr>
<tr>
<td>((A_1^1 - \tau,B_1^1))</td>
<td>NT</td>
<td>MO</td>
</tr>
</tbody>
</table>

Table II.6: Optimal responses of uninformed traders at \( t = 2 \) when the strategy profile at \( t = 1 \) is \((BLO,SLO,NT,NT)\).

**Fourth step.** Given the optimal response of traders at \( t = 2 \), find the optimal action for the rational traders at \( t = 1 \).

Informed traders have no incentives to deviate from the prescribed strategy profile whenever
\[
\delta \frac{1 - \lambda}{2} (\kappa + k_1 - 1) > (\kappa - k_1). \tag{II.8}
\]

Uninformed traders have no incentive to deviate from the prescribed strategy profile whenever
\[
0 \geq (1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)). \tag{II.9}
\]
**Fifth step.** Nobody at \( t = 1 \) has unilateral incentives to deviate from \((BLO, SLO, NT, NT)\) whenever both conditions \((II.8)\) and \((II.9)\) are satisfied. Notice that these conditions can be rewritten as

\[
\sigma < \kappa^{\text{MO-LO}}_1 \quad \text{and} \quad \text{PIN} \geq \psi^{U}_{\text{LO-NT}}.
\]

Finally, Table II.6 includes the moves that are in the equilibrium path at \( t = 2 \) for an uninformed trader, taking into account that \((BLO, SLO, NT, NT)\) is the strategy profile chosen at \( t = 1 \).

**Case B.** Note that when \( k_1 = 1 \) the conditions \((II.2)\) and \((II.7)\) are never satisfied and, therefore, the strategies \((BMO, SMO, BLO, SLO)\) and \((BLO, SLO, BLO, BLO)\) cannot be part of an equilibrium of the game. By contrast, when \( k_1 = 1 \), the conditions \((II.5)\) and \((II.9)\) are always satisfied. However the condition \((II.8)\) is never satisfied when \( k_1 = 1 \) and, therefore, the strategy \((BLO, SLO, NT, NT)\) cannot be either part of an equilibrium of the game.

**Proof of Lemma C.2.** Firstly, we provide the expected profits of each type of order for an informed buyer at \( t = 1 \). We develop our proof for buyers and, by symmetry, the same expressions will follow for sellers.

- The expected profits of a \( BMO \) for an informed buyer are:

\[
\mathbb{E}(\Pi_{BMO,1}^{IH}) = (\kappa - k_1) \tau.
\]

- The expected profits of a \( BLO \) for an informed buyer are:

\[
\mathbb{E}(\Pi_{BLO,1}^{IH}) = \delta \frac{1 - \lambda}{2} (\kappa + k_1 - 1) \tau.
\]

- The expected profits of a \( BDO - BMO \) for an informed buyer are:

  - If \( \theta^{I}_2 \leq \frac{\kappa - k_1}{\kappa} \), then

\[
\mathbb{E}(\Pi_{BDO-BMO,1}^{IH}) = \theta^{I}_1 \kappa \tau + (1 - \theta^{I}_1) \delta \left( \kappa - k_1 + \lambda \left( 1 - \frac{\pi}{2} \right) I^{US,B_1=\emptyset}_{SLO,2} - (k_2 - k_1) \left( \lambda \pi I^{IH,B_1=\emptyset}_{BMO,2} + \frac{1 - \lambda}{2} \right) \right) \tau,
\]

where

\[
I^{US,B_1=\emptyset}_{SLO,2} = \begin{cases} 
1, & \text{if } p^{US,B_1=\emptyset}_{SLO,2} \delta (k_1 - Z\kappa - 1) > 0, \\
0, & \text{otherwise}
\end{cases}
\]

and

\[
I^{IH,B_1=\emptyset}_{BMO,2} = \begin{cases} 
1, & \kappa - k_1 \geq p^{IH,B_1=\emptyset}_{BLO,2} \delta (\kappa + k_1 - 1), \\
0, & \text{otherwise}.
\end{cases}
\]
– If $\frac{\kappa - k_1}{\kappa} < \theta_2^I$, then

$$\mathbb{E}(\Pi^{IH}_{BDO-BMO,1}) = \theta_1^I \kappa \tau + (1 - \theta_1^I) \delta \left( \kappa - k_1 + \lambda \left( \frac{1 - \pi}{2} I^{U_S, B_1=\varnothing}_{SLO,2} + (k_2 - k_1) \frac{1 - \lambda}{2} \right) \right) \tau.$$  

• The expected profits of a $BDO - NT$ for an informed buyer are:

$$\mathbb{E}(\Pi^{IH}_{BDO-NT,1}) = \theta_1^I \kappa \tau.$$  

• The expected profits of $NT$ are:

$$\mathbb{E}(\Pi^{IH}_{NT,1}) = 0.$$  

In what follows, we do not consider the strategies of $NT$ and $BDO - NT$ for the informed traders at $t = 1$ because their corresponding expected profits are always smaller than the expected profits of $BMO$ and $BDO - BMO$, respectively (see Internet Appendix I). Notice that because of this, we write $BDO$ to refer to $BDO - BMO$ for an informed buyer.

Next, we provide the expected profits of each type of orders for an unformed buyer at $t = 1$.

• The expected profits of a $BMO$ for an uninformed buyer are:

$$\mathbb{E}(\Pi^{UB}_{BMO,1}) = -k_1 \tau < 0.$$  

• The expected profits of a $BLO$ for an uninformed buyer are:

$$- \text{If } \theta_2^I \leq \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}}, \text{ then}$$

$$\mathbb{E}(\Pi^{UB}_{BLO,1}) = \frac{\delta}{2} \left( (1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) \right) \tau.$$  

$$- \text{If } \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}} < \theta_2^I, \text{ then}$$

$$\mathbb{E}(\Pi^{UB}_{BLO,1}) = \frac{\delta}{2} (1 - \lambda) (k_1 - 1) \tau.$$  

• The expected profits of a $BDO - BMO$ for an uninformed buyer are:

$$\mathbb{E}(\Pi^{UB}_{BDO-BMO,1}) = (1 - \theta_1^U) \delta \left( \lambda \left( \pi I^{H, B_1=\varnothing}_{SLO,2} + (1 - \pi) I^{U_S, B_1=\varnothing}_{SLO,2} \right) + \left( \frac{1 - \lambda}{2} + \frac{\lambda \pi}{2} I^{H, B_1=\varnothing}_{BMO,2} \right) (k_1 - k_2) - k_1 \tau < 0. \right.$$  

• The expected profits of a $BDO - NT$ for an uninformed buyer are:

$$\mathbb{E}(\Pi^{UB}_{BDO-NT,1}) = 0.$$  

22
• The expected profits of $NT$ are:

$$E(\Pi_{NT,1}^U) = 0.$$ 

In what follows, we do not consider the strategies of $BMO$, $BDO - BMO$, and $BDO - NT$ for the uninformed buyers at $t = 1$ because their corresponding expected profits are always smaller than or equal to the expected profits of $NT$.

We have defined in the paper as $X$ as the uninformed traders’ belief at $t = 2$ about the probability that the $MO$ (observed in the $LOB$) was submitted by an informed trader, $Y$ as the uninformed traders’ belief at $t = 2$ about the probability that the $LO$ (observed in the $LOB$) was submitted by an informed trader, and $Z$ as the uninformed trader’s belief at $t = 2$ about the probability that a $DO$ that returns to the exchange as a $MO$ at the end of the second trading period was submitted by an informed. In each equilibrium $\mathcal{E}_i^D$ we denote by $X^{i,D}$, $Y^{i,D}$, $Z^{i,D}$ the corresponding belief $X, Y, Z$

$$X^{i,D} = \frac{\lambda \pi \Omega_1}{1 - \lambda + \lambda \pi \Omega_1 + \lambda (1 - \pi) \Gamma_1},$$

$$Y^{i,D} = \frac{\pi \Omega_2}{\pi \Omega_2 + (1 - \pi) \Gamma_2},$$

$$Z^{i,D} = \frac{(1 - \theta_i^1)\pi \Omega_3}{(1 - \theta_i^1)\pi \Omega_3 + (1 - \theta_i^1)(1 - \pi) \Gamma_3}.$$ 

(II.12)

In addition, we have defined $P_I$ as the probability of execution of a limit order placed by an informed trader at $t = 2$ when there is no change in the $LOB$ during the first trading period as

$$P_I = p_{BLO,2}^I (B_1 = \emptyset) = p_{SLO,2}^I (B_1 = \emptyset) = \frac{(1 - \theta_i^1)\frac{1}{2} \Gamma_3}{\pi \Omega_3 + (1 - \pi)(\Gamma_0 + \Gamma_3)},$$

and $P_U$ as the probability of execution of a limit order placed by an uninformed trader at $t = 2$ given that there are no changes in prices in the $LOB$ during the first trading period, and equals

$$P_U = p_{BLO,2}^U (B_1 = \emptyset) = p_{SLO,2}^U (B_1 = \emptyset) = \frac{1}{2} \frac{(1 - \theta_i^1)\pi \Omega_3 + (1 - \theta_i^1)(1 - \pi) \Gamma_3}{\pi \Omega_3 + (1 - \pi)(\Gamma_0 + \Gamma_3)}.$$ 

Finally, we have defined the following constants:

$$\theta_{X^{i,D}} = \frac{X^{i,D} \kappa - k_2}{X^{i,D} \kappa - k_2 - k_1},$$

$$\theta_{Y^{i,D}} = \frac{Y^{i,D} \kappa - k_1}{X^{i,D} \kappa - k_1 - \frac{1}{2}},$$

$$\theta = \frac{\kappa - k_1}{\kappa},$$

$$\overline{\theta} = \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}}.$$ 

(II.13)
\( \xi^D_1 : (BMO, SMO, BLO, SLO) \)

**First step.** In this case \( \Omega_0 = 0, \Omega_1 = 1, \Omega_2 = 0, \Omega_3 = 0, \Gamma_0 = 0, \Gamma_1 = 0, \Gamma_2 = 1, \) and \( \Gamma_3 = 0. \n\) Moreover, \( \theta^U_2 = \theta^I_1 \) and \( \theta^L_2 = \theta^I_1. \)

**Second step.** Using Bayes’ rule

\[
X^{1,D} = \frac{\lambda \tau}{1 - \lambda + \lambda \tau}, \quad Y^{1,D} = 0, \quad Z^{1,D} = z \in [0, 1],
\]

\[
p_{UB,B_1=\emptyset}^{UB,B_1=\emptyset} = p_{US,B_1=\emptyset}^{US,B_1=\emptyset} \in [0, 1], \quad \text{and} \quad p_{IB,B_1=\emptyset}^{IB,B_1=\emptyset} = p_{IS,B_1=\emptyset}^{IS,B_1=\emptyset} \in [0, 1].
\]

**Third step.** Using step 2 and taking into account that \( p_{UB,B_1=\emptyset}^{UB} p_{B_1=\emptyset}^{B_1=\emptyset} = p_{US,B_1=\emptyset}^{US} p_{B_1=\emptyset}^{B_1=\emptyset} \in [0, 1], \) at \( t = 2 \) the expected profits of uninformed traders are as follows:

<table>
<thead>
<tr>
<th>( UB )</th>
<th>( BMO )</th>
<th>( BDO )</th>
<th>( BLO )</th>
<th>( NT )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (A^I_1, B^I_1) )</td>
<td>(-k_1 \tau)</td>
<td>0</td>
<td>( \frac{p^{UB,B_1=\emptyset}<em>{BLO}}{p^{UB,B_1=\emptyset}</em>{BLO}} \delta (k_1 - Z^{1,D} - k) \tau )</td>
<td>0</td>
</tr>
<tr>
<td>( (A^I_2, B^I_1) )</td>
<td>( (X^{1,D} - k_2) \tau )</td>
<td>( \theta^U_2 \left( X^{1,D} - \frac{k_2-k_1}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( (A^I_1, B^I_1 + \tau) )</td>
<td>(-k_1 \tau)</td>
<td>( \frac{1}{2} \theta^U_2 \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( (A^I_1, B^I_2) )</td>
<td>(- (X^{1,D} - k_1) \tau )</td>
<td>( \theta^U_2 \left( X^{1,D} - \frac{k_2-k_1}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( (A^I_1 - \tau, B^I_1) )</td>
<td>(- (k_1 - 1) \tau)</td>
<td>( \frac{1}{2} \theta^U_2 \tau )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table II.7: Expected profits of uninformed buyers at \( t = 2 \) when the strategy profile at \( t = 1 \) is \((BMO, SMO, BLO, SLO)\).

<table>
<thead>
<tr>
<th>( US )</th>
<th>( SMO )</th>
<th>( SDO )</th>
<th>( SLO )</th>
<th>( NT )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (A^I_1, B^I_1) )</td>
<td>(-k_1 \tau)</td>
<td>0</td>
<td>( \frac{p^{US,B_1=\emptyset}<em>{SLO}}{p^{US,B_1=\emptyset}</em>{SLO}} \delta (k_1 - Z^{1,D} - k) \tau )</td>
<td>0</td>
</tr>
<tr>
<td>( (A^I_2, B^I_1) )</td>
<td>( - (X^{1,D} - k_1) \tau )</td>
<td>( \theta^U_2 \left( X^{1,D} - \frac{k_2-k_1}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( (A^I_1, B^I_1 + \tau) )</td>
<td>(- (k_1 - 1) \tau)</td>
<td>( \frac{1}{2} \theta^U_2 \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( (A^I_1, B^I_2) )</td>
<td>( (X^{1,D} - k_1) \tau )</td>
<td>( \theta^U_2 \left( X^{1,D} - \frac{k_2-k_1}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( (A^I_1 - \tau, B^I_1) )</td>
<td>(-k_1 \tau)</td>
<td>(- \frac{1}{2} \theta^U_2 \tau )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table II.8: Expected profits of uninformed sellers at \( t = 2 \) when the strategy profile at \( t = 1 \) is \((BMO, SMO, BLO, SLO)\).

Hence, the optimal strategies for the uninformed are:

[Contents continue on the next page]
<table>
<thead>
<tr>
<th>State of the Book</th>
<th>UB</th>
<th>US</th>
</tr>
</thead>
</table>
| \((A_1^1, B_1^1)\) | \(\begin{align*}
NT & \text{ if } \frac{p_{UB, B_1^2} = 0}{p_{BLO, 2} = 0} \text{ or } Z_{1, D}^1 \geq \frac{k_{1-1}}{k} \\
BLO & \text{ if } \frac{p_{UB, B_1^2} = 0}{p_{BLO, 2} > 0} \text{ and } Z_{1, D}^1 < \frac{k_{1-1}}{k}
\end{align*}\) | \(\begin{align*}
NT & \text{ if } \frac{p_{UB, B_1^2} = 0}{p_{BLO, 2} = 0} \text{ or } Z_{1, D}^1 \geq \frac{k_{1-1}}{k} \\
SLO & \text{ if } \frac{p_{UB, B_1^2} = 0}{p_{BLO, 2} > 0} \text{ and } Z_{1, D}^1 < \frac{k_{1-1}}{k}
\end{align*}\) |
| \((A_1^2, B_1^1)\) | \(\begin{align*}
NT & \text{ if } 
BDO & \text{ if } 
BMO & \text{ if }
\frac{k_2-k_1}{2} < X_{1, D_k}^1 \quad \text{and } \theta_{2, D} > \theta_{X_{1, D}}^1
\end{align*}\) | \(\begin{align*}
SLO & \text{ if } \frac{k_2-k_1}{2} < X_{1, D_k}^1
\end{align*}\) |
| \((A_1^1, B_1^1 + \tau)\) | \(NT\) | \(SDO\) |
| \((A_1^1, B_1^2)\) | \(\begin{align*}
BDO & \text{ if } X_{1, D_k}^1 < \frac{k_2-k_1}{2} \\
NT & \text{ if } \frac{k_2-k_1}{2} \leq X_{1, D_k}^1
\end{align*}\) | \(\begin{align*}
SDO & \text{ if } X_{1, D_k}^1 \leq \frac{k_2-k_1}{2} \\
SLO & \text{ if } \frac{k_2-k_1}{2} < X_{1, D_k}^1 \leq k_2 \\
SLO & \text{ if } k_2 < X_{1, D_k}^1 \\
SMO & \text{ if } k_2 < X_{1, D_k}^1 \\
NT & \text{ if } \frac{k_2-k_1}{2} \leq X_{1, D_k}^1
\end{align*}\) |
| \((A_1^1 - \tau, B_1^1)\) | \(BDO\) | \(NT\) |

Table II.9: Optimal strategies of uninformed traders at \(t = 2\) when the strategy profile at \(t = 1\) is \((BMO, SMO, BLO, SLO)\).

Using that \(p_{BLO, 1} = p_{SLO, 1} \in [0, 1]\), at \(t = 2\) the informed traders expected profits are:
\[ (A_1^1, B_1^1) \quad (\kappa - k_1) \quad \tau \quad \theta_1^1 K \tau \quad P_{BLO, 2}^{IH, B_1^1 = \emptyset} \delta (\kappa + k_1 - 1) \tau \quad 0 \]
\[ (A_2^2, B_1^1) \quad (\kappa - k_2) \quad \tau \quad \theta_2^1 \left( \kappa - \frac{k_2 - k_1}{2} \right) \tau \quad 0 \quad 0 \]
\[ (A_1^1, B_1^1 + \tau) \quad (\kappa - k_1) \quad \tau \quad \theta_2^1 \left( \kappa - \frac{1}{2} \right) \tau \quad 0 \quad 0 \]
\[ (A_1^1, B_1^2) \quad (\kappa - k_1) \quad \tau \quad \theta_2^1 \left( \kappa + \frac{k_2 - k_1}{2} \right) \tau \quad 0 \quad 0 \]
\[ (A_1^1 - \tau, B_1^1) \quad (\kappa - k_1 + 1) \quad \tau \quad \theta_2^1 \left( \kappa + \frac{1}{2} \right) \tau \quad 0 \quad 0 \]

Table II.10: Expected profits of informed buyers at \( t = 2 \) when the strategy profile at \( t = 1 \) is \((BMO, SMO, BLO, SLO)\)

\[ (A_1^1, B_1^1) \quad (\kappa - k_1) \quad \tau \quad \theta_1^1 K \tau \quad P_{SLO, 2}^{IL, B_1^1 = \emptyset} \delta (\kappa + k_1 - 1) \tau \quad 0 \]
\[ (A_2^2, B_1^1) \quad (\kappa - k_2) \quad \tau \quad \theta_2^1 \left( \kappa + \frac{k_2 - k_1}{2} \right) \tau \quad 0 \quad 0 \]
\[ (A_1^1, B_1^1 + \tau) \quad (\kappa - k_1 + 1) \quad \tau \quad \theta_2^1 \left( \kappa + \frac{1}{2} \right) \tau \quad 0 \quad 0 \]
\[ (A_1^1, B_1^2) \quad (\kappa - k_2) \quad \tau \quad \theta_2^1 \left( \kappa + \frac{k_1 - k_2}{2} \right) \tau \quad 0 \quad 0 \]
\[ (A_1^1 - \tau, B_1^1) \quad (\kappa - k_1) \quad \tau \quad \theta_2^1 \left( \kappa + \frac{1}{2} \right) \tau \quad 0 \quad 0 \]

Table II.11: Expected profits of informed sellers at \( t = 2 \) when the strategy profile at \( t = 1 \) is \((BMO, SMO, BLO, SLO)\)

Define \( BX, SX, BY, SY \) as

\[
BX = \begin{cases} 
BMO & \text{if } P_{BLO, 2}^{IH, B_1^1 = \emptyset} \leq \frac{\kappa - k_1}{\delta (\kappa + k_1 - 1)} \\
BLO & \text{if } P_{BLO, 2}^{IH, B_1^1 = \emptyset} > \frac{\kappa - k_3}{\delta (\kappa + k_1 - 1)},
\end{cases}
\]

\[
SX = \begin{cases} 
SMO & \text{if } P_{SLO, 2}^{IL, B_1^1 = \emptyset} \leq \frac{\kappa - k_1}{\delta (\kappa + k_1 - 1)} \\
SLO & \text{if } P_{SLO, 2}^{IL, B_1^1 = \emptyset} > \frac{\kappa - k_1}{\delta (\kappa + k_1 - 1)},
\end{cases}
\]

\[
BY = \begin{cases} 
BDO & \text{if } P_{BLO, 2}^{IH, B_1^1 = \emptyset} < \frac{\theta_2^1 \kappa}{\delta (\kappa + k_1 - 1)} \\
BLO & \text{if } P_{BLO, 2}^{IH, B_1^1 = \emptyset} \geq \frac{\theta_2^1 \kappa}{\delta (\kappa + k_1 - 1)},
\end{cases}
\]

\[
SY = \begin{cases} 
SDO & \text{if } P_{BLO, 2}^{IH, B_1^1 = \emptyset} < \frac{\theta_2^1 \kappa}{\delta (\kappa + k_1 - 1)} \\
SLO & \text{if } P_{BLO, 2}^{IH, B_1^1 = \emptyset} \geq \frac{\theta_2^1 \kappa}{\delta (\kappa + k_1 - 1)},
\end{cases}
\]

The optimal strategy for an informed trader at \( t = 2 \) is:
Given the optimal response of traders at $t = 2$, we find the optimal action for the traders at $t = 1$ in each of the 6 cases. However, given the nature of this particular equilibrium, we can group cases and analyze them in the following way:

Case $I_1 + I_2 + I_3 : \theta_2^1 \leq \frac{\kappa - k_1}{\kappa}$

<table>
<thead>
<tr>
<th>Condition</th>
<th>Optimal Strategies of Informed Traders at $t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_2^1 \leq \frac{\kappa - k_2}{\kappa - \frac{2k_1 - 1}{2}}$</td>
<td>(Case $I_1$)</td>
</tr>
<tr>
<td>$\frac{\kappa - k_2}{\kappa - \frac{2k_1 - 1}{2}} &lt; \theta_2^1 \leq \frac{\kappa - k_1}{\kappa + \frac{2k_1 - 1}{2}}$</td>
<td>(Case $I_2$)</td>
</tr>
<tr>
<td>$\frac{\kappa - k_1}{\kappa + \frac{2k_1 - 1}{2}} &lt; \theta_2^1 \leq \frac{\kappa - k_1}{\kappa}$</td>
<td>(Case $I_3$)</td>
</tr>
<tr>
<td>$\frac{\kappa - k_1}{\kappa} &lt; \theta_2^1 \leq \frac{\kappa - k_1 - 2}{\kappa - 2}$</td>
<td>(Case $I_4$)</td>
</tr>
<tr>
<td>$\frac{\kappa - k_1}{\kappa - 2} &lt; \theta_2^1 \leq \frac{\kappa - k_1 + 1}{\kappa + 2}$</td>
<td>(Case $I_5$)</td>
</tr>
<tr>
<td>$\frac{\kappa - k_1 + 1}{\kappa + 2} &lt; \theta_2^1$</td>
<td>(Case $I_6$)</td>
</tr>
</tbody>
</table>

Table II.12: Optimal strategies of informed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BMO, SMO, BLO, SLO)$
• Informed traders

As \( \theta_2^I \leq \frac{\kappa - k_1}{\kappa} \), informed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever

\[
\kappa - k_1 \geq \frac{1 - \lambda}{2} \delta (\kappa + k_1 - 1) \quad \text{and}
\]

\[
\kappa - k_1 \geq \theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 + \frac{1 - \pi}{2} I_{SLO,2}^{US,B_1=\emptyset} - (k_2 - k_1) \left( \lambda \pi I_{BMO,2}^{IH,B_1=\emptyset} + \frac{1 - \lambda}{2} \right) \right).
\]

• Uninformed traders

As \( \theta_2^I \leq \frac{\kappa - k_1}{\kappa} \leq \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}} \), uninformed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever

\[
(1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) > 0.
\]

Case \( I_4 + I_5 + I_6 : \frac{\kappa - k_1}{\kappa} < \theta_2^I \)

• Informed traders

Consider an informed buyer at \( t = 1 \). If he chooses a \( BMO \), then he obtains

\[
E \left( \Pi_{BMO,1}^{IH} \right) = (\kappa - k_1) \tau.
\]

If instead he deviates towards a \( BDO \), he will obtain

\[
E \left( \Pi_{BDO,1}^{IH} \right) = \theta_1^I \kappa \tau + (1 - \theta_1^I) \delta \left[ \frac{1 - \pi}{2} I_{SLO,2}^{US,B_1=\emptyset} + (\kappa - k_1) - (k_2 - k_1) \left( \lambda \pi I_{BMO,2}^{IH,B_1=\emptyset} + \frac{1 - \lambda}{2} \right) \right] \tau.
\]

Combining the previous expression and the fact that \( \frac{\kappa - k_1}{\kappa} < \theta_2^I = \theta_1^I \), it follows that

\[
E \left( \Pi_{BDO,1}^{IH} \right) > E \left( \Pi_{BMO,1}^{IH} \right)
\]

is always satisfied and, hence, we conclude that in this case there is no equilibrium in which \((BMO,SMO,BLO,SLO)\) is the strategy profile chosen at \( t = 1 \).
**Fifth step.** Based on the above, nobody at \( t = 1 \) has unilateral incentives to deviate whenever

\[
\theta^I_1 \leq \frac{\kappa - k_1}{\kappa},
\]

\[
(1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) > 0,
\]

\[
\kappa - k_1 \geq \frac{1 - \lambda}{2} \delta (\kappa + k_1 - 1)
\]

and

\[
\kappa - k_1 \geq (1 - \theta^I_1) \delta \left( \lambda \left( \frac{1 - \pi}{2} \right) I_{SLO,2}^{US,B_1=\emptyset} + (\kappa - k_1) - (k_2 - k_1) \left( \lambda \pi I_{BMO,2}^{IH,B_1=\emptyset} + \frac{1 - \lambda}{2} \right) \right).
\]

These conditions are equivalent to

\[
\theta^I_1 \leq \hat{\theta},
\]

\[
PIN < \psi^U_{LO-NT},
\]

\[
\sigma \geq \kappa^I_{MO-LO},
\]

and

\[
\theta^I_1 \leq \hat{\theta}_{MO-DO},
\]

where \( \hat{\theta} \) is defined in (II.13) and

\[
\hat{\theta}_{MO-DO} \equiv \frac{\kappa - k_1 - \delta \left( \lambda \left( \frac{1 - \pi}{2} \right) I_{SLO,2}^{US,B_1=\emptyset} + (\kappa - k_1) - (k_2 - k_1) \left( \lambda \pi I_{BMO,2}^{IH,B_1=\emptyset} + \frac{1 - \lambda}{2} \right) \right)}{\kappa - \delta \left( \lambda \left( \frac{1 - \pi}{2} \right) I_{SLO,2}^{US,B_1=\emptyset} + (\kappa - k_1) - (k_2 - k_1) \left( \lambda \pi I_{BMO,2}^{IH,B_1=\emptyset} + \frac{1 - \lambda}{2} \right) \right)}.
\]

Notice that it can be proved that \( \hat{\theta}_{MO-DO} \leq \hat{\theta} \) and, therefore, we can simplify further to

\[
\sigma \geq \kappa^I_{MO-LO},
\]

\[
PIN < \psi^U_{LO-NT}
\]

and

\[
\theta^I_1 \leq \hat{\theta}_{MO-DO}.
\]

(II.14)

Finally, in the following tables we include the moves that are in the equilibrium path taking into account the conditions that must be satisfied if \((BMO, SMO, BLO, SLO)\) is the strategy profile chosen at \( t = 1 \) and the fact that in this case \( \theta^I_2 = \theta^I_1 \).

Concerning uninformed traders notice that the condition \((1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) > 0 \) implies that \( X^{1-D} \kappa < k_1 - 1 < k_2 \). Hence, the optimal choice of uninformed traders is
Table II.13: Optimal choice of uninformed traders at \( t = 2 \) when the strategy profile at \( t = 1 \) is \((BMO,SMO,BLO,SLO)\)

In relation to informed traders:

Table II.14: Optimal choice of informed traders at \( t = 2 \) when the strategy profile at \( t = 1 \) is \((BMO,SMO,BLO,SLO)\)
\[ \mathcal{E}_2^D : (BMO, SMO, NT, NT) \]

**First step.** In this case \( \Omega_0 = 0, \Omega_1 = 1, \Omega_2 = 0, \Omega_3 = 0, \Gamma_0 = 1, \Gamma_1 = 0, \Gamma_2 = 0, \) and \( \Gamma_3 = 0. \) Moreover, \( \theta_1^I = \theta_1^U \) and \( \theta_2^I = \theta_2^U. \)

**Second step.** Using Bayes’ rule

\[
X_2^D = \frac{\lambda \pi}{1 - \lambda + \lambda \pi}, \quad Y_2^D = y \in [0, 1], \quad Z_2^D = z \in [0, 1],
\]

\[ p_{UB, B_1 = \emptyset} = p_{US, B_1 = \emptyset} = 0, \quad \text{and} \quad p_{IB, B_1 = \emptyset} = p_{IL, B_1 = \emptyset} = 0. \]

**Third step.** Using step 2 and taking into account that \( p_{UB, B_1 = \emptyset} = 0, \) at \( t = 2, \) the expected profits of uninformed buyers are as follows:

<table>
<thead>
<tr>
<th>UB</th>
<th>BMO</th>
<th>BDO</th>
<th>BLO</th>
<th>NT</th>
</tr>
</thead>
<tbody>
<tr>
<td>((A_1^i, B_1^i))</td>
<td>(-k_1 \tau)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>((A_2^i, B_1^i))</td>
<td>((X_2^D, k_2) \tau)</td>
<td>(\theta_2^U \left( X_2^D, k_2, -\frac{k_2 - k_1}{2} \right) \tau)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>((A_1^i, B_1^i + \tau))</td>
<td>((Y_2^D, k_1) \tau)</td>
<td>(\theta_2^U \left( Y_2^D, k_1 \right) \tau)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>((A_1^i, B_2^i))</td>
<td>(-k_2 \tau)</td>
<td>(-\theta_2^U \left( X_2^D, k_2, -\frac{k_2 - k_1}{2} \right) \tau)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>((A_1^i - \tau, B_1^i))</td>
<td>(-k_1 \tau)</td>
<td>(-\theta_2^U \left( Y_2^D, k_1 \right) \tau)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table II.15: Expected profits of uninformed buyers at \( t = 2 \) when the strategy profile at \( t = 1 \) is \( (BMO, SMO, NT, NT) \)

By symmetry, we can find the expected profits of uninformed sellers. Hence, the optimal strategies for the uninformed traders are:

31
### Optimal Strategies of Uninformed Traders at $t = 2$

<table>
<thead>
<tr>
<th>State of the Book</th>
<th>UB</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(A_1^i, B_1^i)$</td>
<td>NT</td>
<td>NT</td>
</tr>
</tbody>
</table>
| $(A_2^i, B_1^i)$ | $\begin{cases} 
NT & \text{if } X^{2,D_k} \leq \frac{k_2-k_1}{2} \\
BDO & \text{if } \frac{k_2-k_1}{2} < X^{2,D_k} \leq k_2 \\
BDO & \text{if } k_2 < X^{2,D_k} \\
BMO & \text{if } k_2 < X^{2,D_k} \\
\end{cases}$ | $\begin{cases} 
SDO & \text{if } X^{2,D_k} < \frac{k_2-k_1}{2} \\
NT & \text{if } \frac{k_2-k_1}{2} \leq X^{2,D_k} \\
\end{cases}$ |
| $(A_1^i, B_1^i + \tau)$ | $\begin{cases} 
NT & \text{if } Y^{2,D_K} \leq \frac{1}{2} \\
BDO & \text{if } \frac{1}{2} < Y^{2,D_K} < k_1 \\
BDO & \text{if } Y^{2,D_K} \geq k_1 \\
BMO & \text{if } Y^{2,D_K} \geq k_1 \\
\end{cases}$ | $\begin{cases} 
SDO & \text{if } Y^{2,D_K} < \frac{1}{2} \\
NT & \text{if } \frac{1}{2} \leq Y^{2,D_K} \\
\end{cases}$ |
| $(A_1^i, B_1^i)$ | $\begin{cases} 
BDO & \text{if } X^{2,D_K} < \frac{k_2-k_1}{2} \\
NT & \text{if } \frac{k_2-k_1}{2} \leq X^{2,D_K} \\
\end{cases}$ | $\begin{cases} 
SDO & \text{if } X^{2,D_K} \leq \frac{k_2-k_1}{2} \\
SDO & \text{if } \frac{k_2-k_1}{2} < X^{2,D_K} \leq k_2 \\
SDO & \text{if } k_2 < X^{2,D_K} \\
SMO & \text{if } k_2 < X^{2,D_K} \\
\end{cases}$ |
| $(A_1^i - \tau, B_1^i)$ | $\begin{cases} 
BDO & \text{if } Y^{2,D_K} < \frac{1}{2} \\
NT & \text{if } \frac{1}{2} \leq Y^{2,D_K} \\
\end{cases}$ | $\begin{cases} 
SDO & \text{if } Y^{2,D_K} < \frac{1}{2} \\
SDO & \text{if } \frac{1}{2} \leq Y^{2,D_K} \\
SMO & \text{if } Y^{2,D_K} \geq k_1 \\
\end{cases}$ |

Table II.16: Optimal strategies of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BMO, SMO, NT, NT)$

Concerning the informed buyers, using $p_{BLO,2} = 0$, their expected profits at $t = 2$ are as follows:

<table>
<thead>
<tr>
<th>$IH$</th>
<th>$BMO$</th>
<th>$BDO$</th>
<th>$BLO$</th>
<th>NT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(A_1^i, B_1^i)$</td>
<td>$(\kappa - k_1) \tau$</td>
<td>$\theta_2^{\kappa \tau}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(A_2^i, B_1^i)$</td>
<td>$(\kappa - k_2) \tau$</td>
<td>$\theta_2^{\kappa - k_2 - k_1} \tau$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(A_1^i, B_1^i + \tau)$</td>
<td>$(\kappa - k_1) \tau$</td>
<td>$\theta_2^{\kappa - \frac{k_2-k_1}{2}} \tau$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(A_1^i, B_1^i)$</td>
<td>$(\kappa - k_1) \tau$</td>
<td>$\theta_2^{\kappa - \frac{k_2-k_1}{2}} \tau$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(A_1^i - \tau, B_1^i)$</td>
<td>$(\kappa - k_1 + 1) \tau$</td>
<td>$\theta_2^{\kappa + \frac{k_2-k_1}{2}} \tau$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table II.17: Expected profits of informed buyers at $t = 2$ when the strategy profile at $t = 1$ is $(BMO, SMO, NT, NT)$
By symmetry, we can find the expected profits of informed sellers at $t = 2$. Hence, the optimal strategies for informed traders at $t = 2$ are given in the following table:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Optimal Strategies of Informed Traders at $t = 2$</th>
<th>State of the Book</th>
<th>IH</th>
<th>IL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case $I_1$</td>
<td>$\frac{\kappa - k_2}{\kappa - k_1}$ $\leq \theta_2^I$</td>
<td>$(A_1^1, B_1^1)$</td>
<td>$BMO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_2^1, B_1^1)$</td>
<td>$BMO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_2^1, B_1^1 + \tau)$</td>
<td>$BMO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1, B_1^2)$</td>
<td>$BMO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1, B_1^1 - \tau, B_1^1)$</td>
<td>$BMO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td>Case $I_2$</td>
<td>$\frac{\kappa - k_2}{\kappa - k_1}$ $&lt; \theta_2^I \leq \frac{\kappa - k_3}{\kappa - k_2}$</td>
<td>$(A_1^1, B_1^1)$</td>
<td>$BMO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_2^1, B_1^1)$</td>
<td>$BDO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_2^1, B_1^1 + \tau)$</td>
<td>$BMO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1, B_1^2)$</td>
<td>$BDO$</td>
<td>$SDO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1 - \tau, B_1^1)$</td>
<td>$BMO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td>Case $I_3$</td>
<td>$\frac{\kappa - k_3}{\kappa - k_2 - k_1}$ $&lt; \theta_2^I \leq \frac{\kappa - k_1}{\kappa}$</td>
<td>$(A_1^1, B_1^1)$</td>
<td>$BMO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_2^1, B_1^1)$</td>
<td>$BDO$</td>
<td>$SDO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_2^1, B_1^1 + \tau)$</td>
<td>$BMO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1, B_1^2)$</td>
<td>$BDO$</td>
<td>$SDO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1 - \tau, B_1^1)$</td>
<td>$BMO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td>Case $I_4$</td>
<td>$\frac{\kappa - k_1}{\kappa} &lt; \theta_2^I \leq \frac{\kappa - k_1}{\kappa - \frac{1}{2}}$</td>
<td>$(A_1^1, B_1^1)$</td>
<td>$BDO$</td>
<td>$SDO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_2^1, B_1^1)$</td>
<td>$BDO$</td>
<td>$SDO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1, B_1^1 + \tau)$</td>
<td>$BMO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1, B_1^2)$</td>
<td>$BDO$</td>
<td>$SDO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1 - \tau, B_1^1)$</td>
<td>$BMO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td>Case $I_5$</td>
<td>$\frac{\kappa - k_1}{\kappa - \frac{1}{2}} &lt; \theta_2^I \leq \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}}$</td>
<td>$(A_1^1, B_1^1)$</td>
<td>$BDO$</td>
<td>$SDO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_2^1, B_1^1)$</td>
<td>$BDO$</td>
<td>$SDO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1, B_1^1 + \tau)$</td>
<td>$BDO$</td>
<td>$SMO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1, B_1^2)$</td>
<td>$BDO$</td>
<td>$SDO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1 - \tau, B_1^1)$</td>
<td>$BMO$</td>
<td>$SDO$</td>
</tr>
<tr>
<td>Case $I_6$</td>
<td>$\frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}} &lt; \theta_2^I$</td>
<td>$(A_1^1, B_1^1)$</td>
<td>$BDO$</td>
<td>$SDO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_2^1, B_1^1)$</td>
<td>$BDO$</td>
<td>$SDO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1, B_1^1 + \tau)$</td>
<td>$BDO$</td>
<td>$SDO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1, B_1^2)$</td>
<td>$BDO$</td>
<td>$SDO$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_1^1 - \tau, B_1^1)$</td>
<td>$BDO$</td>
<td>$SDO$</td>
</tr>
</tbody>
</table>

Table II.18: Optimal strategies of informed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BMO, SMO, NT, NT)$
Fourth step. Given the optimal response of traders at \( t = 2 \), find the optimal action for the traders at \( t = 1 \) in each of the 6 cases. However, given the nature of this particular equilibrium, we can group cases and analyze them as follows:

Case \( I_1 + I_2 + I_3 : \theta_2^I \leq \frac{\kappa - k_1}{\kappa} \)

- **Informed traders**

  As \( \theta_2^I \leq \frac{\kappa - k_1}{\kappa} \), informed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever

  \[
  \kappa - k_1 \geq 1 - \frac{\lambda}{2} \delta (\kappa + k_1 - 1) \quad \text{and} \quad \kappa - k_1 \geq \theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1 - \lambda}{2} \right) \right) \tau,
  \]

  since \( I_{SLO,2}^{US,B} = \emptyset \) and \( I_{BMO,2}^{IH,B} = \emptyset \).

- **Uninformed traders**

  As \( \theta_2^I \leq \frac{\kappa - k_1}{\kappa} \leq \frac{\kappa - k_1 + 1}{\kappa + 1} \), uninformed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever

  \[
  0 \geq (1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)).
  \]

Case \( I_4 + I_5 + I_6 : \frac{\kappa - k_1}{\kappa} < \theta_2^I \)

- **Informed traders**

  Consider an informed buyer at \( t = 1 \). If he chooses a \( BMO \), then he will obtain

  \[
  E(\Pi_{BMO,1}^{IH}) = (\kappa - k_1) \tau.
  \]

  If instead he deviates towards a \( BDO \), he will obtain

  \[
  E(\Pi_{BDO,1}^{IH}) = \theta_1^I \kappa \tau + (1 - \theta_1^I) \delta \left[ (\kappa - k_1) - (k_2 - k_1) \left( \lambda \pi I_{BMO,2}^{IH,B} = \emptyset + \frac{1 - \lambda}{2} \right) \right] \tau.
  \]

  Combining the previous expression and the fact that \( \frac{\kappa - k_1}{\kappa} < \theta_2^I = \theta_1^I \), it follows that

  \[
  E(\Pi_{BDO,1}^{IH}) > E(\Pi_{BMO,1}^{IH})
  \]

  is always satisfied and, hence, in this case we conclude that in this case there is no equilibrium in which \((BMO, SMO, NT, NT)\) is the strategy profile chosen at \( t = 1 \).
**Fifth step.** Based on the above, nobody at $t = 1$ has unilateral incentives to deviate whenever

$$
\theta_1^l \leq \frac{\kappa - k_1}{\kappa},
$$

$$(1 - \lambda)(k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) \leq 0,$$

$$\kappa - k_1 \geq \frac{1 - \lambda}{2} \delta (\kappa + k_1 - 1) \text{ and}$$

$$\kappa - k_1 \geq \theta_1^l \kappa + (1 - \theta_1^l) \delta \left( \kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1 - \lambda}{2} \right) \right).$$

These conditions are equivalent to:

$$\theta_1^l \leq \bar{\theta},$$

$$PIN \geq \psi^U_{LO-NT},$$

$$\sigma \geq \kappa^{I}_{MO-LO \tau} \text{, and}$$

$$\theta_1^l \leq \theta_{MO-DO},$$

where $\bar{\theta}$ is defined in \textit{(II.13)} and

$$\bar{\theta}_{MO-DO} \equiv \frac{\kappa - k_1 - \delta (\kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1 - \lambda}{2} \right))}{\kappa - \delta (\kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1 - \lambda}{2} \right))}. \hspace{1cm} \textbf{(II.15)}$$

Notice also that it can be easily proved that $\bar{\theta}_{MO-DO} \leq \bar{\theta}$ and, therefore, we can simplify further to

$$\sigma \geq \kappa^{I}_{MO-LO \tau}, \hspace{0.2cm} PIN \geq \psi^U_{LO-NT}, \hspace{0.2cm} \text{and} \hspace{0.2cm} \theta_1^l \leq \bar{\theta}_{MO-DO}. \hspace{1cm} \textbf{(II.16)}$$

Finally, in the following tables we include the decisions that are in the equilibrium path taking into account the conditions that must be satisfied if $(BMO, SMO, NT, NT)$ is the strategy profile chosen at $t = 1$ and that in this case $\theta_2^l = \theta_1^l$.

In relation to uninformed traders, and taking into account that a necessary condition for this equilibrium tells us that $k_1 - 1 \leq X^{2,D}_2 \kappa$, the following cases can be distinguished:
### Condition

<table>
<thead>
<tr>
<th>Case $U_1^{2, D}$</th>
<th>$k_1 - 1 &lt; X^{2, D, \kappa} &lt; \frac{k_2 - k_1}{2}$</th>
<th>State of the Book</th>
<th>UB</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(A_1^1, B_1^1)$</td>
<td>NT</td>
<td>NT</td>
<td>NT</td>
</tr>
<tr>
<td></td>
<td>$(A_2^1, B_1^1)$</td>
<td>NT</td>
<td>SDO</td>
<td>NT</td>
</tr>
<tr>
<td></td>
<td>$(A_1^1, B_1^2)$</td>
<td>BDO</td>
<td>NT</td>
<td>NT</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case $U_2^{2, D}$</th>
<th>$k_1 - 1 &lt; X^{2, D, \kappa} = \frac{k_2 - k_1}{2}$</th>
<th>State of the Book</th>
<th>UB</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(A_1^1, B_1^1)$</td>
<td>NT</td>
<td>NT</td>
<td>NT</td>
</tr>
<tr>
<td></td>
<td>$(A_2^2, B_1^1)$</td>
<td>NT</td>
<td>NT</td>
<td>NT</td>
</tr>
<tr>
<td></td>
<td>$(A_1^1, B_1^2)$</td>
<td>NT</td>
<td>NT</td>
<td>NT</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case $U_3^{2, D}$</th>
<th>$\max\left{ k_1 - 1, \frac{k_2 - k_1}{2} \right} &lt; X^{2, D, \kappa} \leq k_2$ or $k_2 &lt; X^{2, D, \kappa}$ and $\theta^U_2 &gt; \theta_{X^2, D}$</th>
<th>State of the Book</th>
<th>UB</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(A_1^1, B_1^1)$</td>
<td>NT</td>
<td>NT</td>
<td>NT</td>
</tr>
<tr>
<td></td>
<td>$(A_2^2, B_1^1)$</td>
<td>BDO</td>
<td>NT</td>
<td>NT</td>
</tr>
<tr>
<td></td>
<td>$(A_1^1, B_1^2)$</td>
<td>NT</td>
<td>SDO</td>
<td>NT</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case $U_4^{2, D}$</th>
<th>$k_2 &lt; X^{2, D, \kappa}$ and $\theta^U_2 \leq \theta_{X^2, D}$</th>
<th>State of the Book</th>
<th>UB</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(A_1^1, B_1^1)$</td>
<td>NT</td>
<td>NT</td>
<td>NT</td>
</tr>
<tr>
<td></td>
<td>$(A_2^2, B_1^1)$</td>
<td>BMO</td>
<td>NT</td>
<td>NT</td>
</tr>
<tr>
<td></td>
<td>$(A_1^1, B_1^2)$</td>
<td>NT</td>
<td>SMO</td>
<td>NT</td>
</tr>
</tbody>
</table>

**Table II.19:** Optimal choice of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BMO, SMO, NT, NT)$

In relation to informed traders:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Optimal Choice of Informed Traders at $t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State of the Book</td>
</tr>
<tr>
<td>Case $I_1$</td>
<td>$\theta^I_1 \leq \frac{\kappa - k_2}{\kappa - \frac{k_2 - k_1}{2}}$</td>
</tr>
<tr>
<td></td>
<td>$(A_2^1, B_1^1)$</td>
</tr>
<tr>
<td></td>
<td>$(A_1^1, B_2^1)$</td>
</tr>
<tr>
<td>Case $I_2$</td>
<td>$\frac{\kappa - k_2}{\kappa - \frac{k_2 - k_1}{2}} &lt; \theta^I_1 \leq \frac{\kappa - k_1}{\kappa + \frac{k_2 - k_1}{2}}$</td>
</tr>
<tr>
<td></td>
<td>$(A_2^1, B_1^1)$</td>
</tr>
<tr>
<td></td>
<td>$(A_1^1, B_2^1)$</td>
</tr>
<tr>
<td>Case $I_3$</td>
<td>$\frac{\kappa - k_1}{\kappa + \frac{k_2 - k_1}{2}} &lt; \theta^I_1 \leq \frac{\kappa - k_1}{\kappa}$</td>
</tr>
<tr>
<td></td>
<td>$(A_2^1, B_1^1)$</td>
</tr>
<tr>
<td></td>
<td>$(A_1^1, B_2^1)$</td>
</tr>
</tbody>
</table>

**Table II.20:** Optimal choice of informed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BMO, SMO, NT, NT)$
\( \mathcal{E}_3^D : (BLO, SLO, BLO, BLO) \)

**First step.** In this case \( \Omega_0 = 0, \Omega_1 = 0, \Omega_2 = 1, \Omega_3 = 0, \Gamma_0 = 0, \Gamma_1 = 0, \Gamma_2 = 1, \) and \( \Gamma_3 = 0. \) Moreover, \( \theta^1_1 = \theta^1_2 \) and \( \theta^1_1 = \theta^1_2. \)

**Second step.** Using Bayes’ rule,

\[
X^{3,D} = 0, \ Y^{3,D} = \pi \text{ and } Z^{3,D} = z \in [0, 1],
\]

\[
P_{BLO,2}^{UB,B_1=\emptyset} = p_{SLO,2}^{UB,B_1=\emptyset} \in [0, 1], \text{ and } p_{BLO,2}^{UB,B_1=\emptyset} = p_{SLO,2}^{UB,B_1=\emptyset} \in [0, 1].
\]

**Third step.** Using step 2, at \( t = 2 \), the expected profits of uninformed buyers are as follows:

<table>
<thead>
<tr>
<th>( UB )</th>
<th>( BMO )</th>
<th>( BDO )</th>
<th>( BLO )</th>
<th>( NT )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (A^1_1, B^1_1) )</td>
<td>(-k_1 \tau )</td>
<td>0</td>
<td>(-\theta^1_2 \tau )</td>
<td>0</td>
</tr>
<tr>
<td>( (A^1_2, B^1_2) )</td>
<td>(-k_2 \tau )</td>
<td>(-\theta^1_2 \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( (A^1_1, B^1_1 + \tau) )</td>
<td>(Y^{3,D} - k_1 ) ( \tau )</td>
<td>(\theta^1_2 ) ( \left(Y^{3,D} - \frac{k_1}{\tau} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( (A^1_1, B^1_2) )</td>
<td>(-k_1 \tau )</td>
<td>(\theta^1_2 ) ( \left(Y^{3,D} - \frac{k_1}{\tau} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( (A^1_1 - \tau, B^1_1) )</td>
<td>(Y^{3,D} - k_1 ) ( \tau )</td>
<td>(-\theta^1_2 ) ( \left(Y^{3,D} - \frac{k_1}{\tau} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table II.21: Expected profits of uninformed buyers at \( t = 2 \) when the strategy profile at \( t = 1 \) is \( (BLO, SLO, BLO, SLO) \).

By symmetry, we can find the expected profits of uninformed sellers. Hence, the optimal strategies for the uninformed traders are:

<table>
<thead>
<tr>
<th>State of the Book</th>
<th>( UB )</th>
<th>( US )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (A^1_1, B^1_1) )</td>
<td>{ ( NT ) if ( UB,B_1=\emptyset ) ( = 0 ) or ( Z^{3,D} \geq k_1 ) ( - 1 ) ( ), ( BLO ) if ( UB,B_1=\emptyset ) ( &gt; 0 ) ( ) and ( Z^{3,D} &lt; k_1 ) ( - 1 ) ( ) }</td>
<td>{ ( NT ) if ( UB,B_1=\emptyset ) ( = 0 ) or ( Z^{3,D} \geq k_1 ) ( - 1 ) ( ), ( SLO ) if ( UB,B_1=\emptyset ) ( &gt; 0 ) ( ) and ( Z^{3,D} &lt; k_1 ) ( - 1 ) ( ) }</td>
</tr>
<tr>
<td>( (A^1_1, B^1_1 + \tau) )</td>
<td>( NT ) if ( Y^{3,D} \leq \frac{1}{2} ), ( BDO ) if ( \frac{1}{2} &lt; Y^{3,D} \leq k_1 ) ( ), ( BDO ) if ( Y^{3,D} &gt; k_1 ) ( ) and ( \theta^1_2 &gt; \theta ), ( BMO ) if ( Y^{3,D} &gt; k_1 ) ( ) and ( \theta^1_2 \leq \theta )</td>
<td>{ ( SDO ) if ( Y^{3,D} &lt; \frac{1}{2} ), ( NT ) if ( Y^{3,D} \geq \frac{1}{2} ) ( ) }</td>
</tr>
<tr>
<td>( (A^1_1, B^1_2) )</td>
<td>( BDO )</td>
<td>( NT )</td>
</tr>
<tr>
<td>( (A^1_1 - \tau, B^1_1) )</td>
<td>{ ( BDO ) if ( Y^{3,D} \leq \frac{1}{2} ), ( NT ) if ( Y^{3,D} \geq \frac{1}{2} ) ( ) }</td>
<td>{ ( SDO ) if ( Y^{3,D} &lt; \frac{1}{2} ), ( SDO ) if ( Y^{3,D} \leq \frac{1}{2} ), ( SDO ) if ( Y^{3,D} \geq \frac{1}{2} ) ( ), ( SMO ) if ( Y^{3,D} &gt; \theta ) ( ) and ( \theta^1_2 &gt; \theta ) }</td>
</tr>
</tbody>
</table>

Table II.22: Optimal strategies of uninformed traders at \( t = 2 \) when the strategy profile at \( t = 1 \) is \( (BLO, SLO, BLO, SLO) \).
Concerning informed traders, as before, the optimal strategies for these traders at \( t = 2 \) are given in Table II.12.

**Fourth step.** Given the optimal responses of traders at \( t = 2 \), find the optimal action for the traders at \( t = 1 \) in each of the 6 cases. However, given the nature of this particular equilibrium, we can group cases into the following and analyze them:

Case I:

\[ I_1 + I_2 + I_3 : \theta_2^I \leq \frac{\kappa - k_1}{\kappa} \]

- **Informed traders**

As \( \theta_2^I \leq \frac{\kappa - k_1}{\kappa} \), informed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever

\[
\begin{align*}
\delta \frac{1 - \lambda}{2} (\kappa + k_1 - 1) &> \kappa - k_1 \quad \text{and} \\
\delta \frac{1 - \lambda}{2} (\kappa + k_1 - 1) &\geq \theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 + \lambda \frac{(1 - \pi)}{2} I_{US,B_1 = \emptyset}^{US,B_1 = \emptyset} \right. \\
&\left. - (k_2 - k_1) \left( \lambda \pi I_{BM,B_2 = \emptyset}^{BM,B_2 = \emptyset} + \frac{1 - \lambda}{2} \right) \right).
\end{align*}
\]

- **Uninformed traders**

As \( \theta_2^I \leq \frac{\kappa - k_1}{\kappa} \leq \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}} \), uninformed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever

\[
(1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) > 0.
\]

**Fifth step.** From the previous two inequalities, nobody at \( t = 1 \) has unilateral incentives to deviate whenever

\[
(1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) > 0,
\]

\[
\delta \frac{1 - \lambda}{2} (\kappa + k_1 - 1) > \kappa - k_1, \quad \text{and}
\]

\[
\delta \frac{1 - \lambda}{2} (\kappa + k_1 - 1) \geq \theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 + \lambda \frac{(1 - \pi)}{2} I_{US,B_1 = \emptyset}^{US,B_1 = \emptyset} \right. \\
\left. - (k_2 - k_1) \left( \lambda \pi I_{BM,B_2 = \emptyset}^{BM,B_2 = \emptyset} + \frac{1 - \lambda}{2} \right) \right).
\]

These conditions are therefore equivalent to

\[
\begin{align*}
PIN &< \psi_{LO-NT}, \\
\sigma &< \kappa_{MO-LO}^{\tau}, \quad \text{and} \\
\theta_1^I &\leq \min \{ \theta, \theta_{LO-DO} \},
\end{align*}
\]

where \( \theta \) is defined in (II.13) and
\[ \theta_{LO-DO} = \frac{\delta^{1-\lambda} (\kappa + k_1 - 1) - \delta \left( \kappa - k_1 + \frac{1-\pi}{2} I_{SLO,2}^{B1=\emptyset} - (k_2 - k_1) \left( \lambda \pi I_{BMO,2}^{I_{B1=\emptyset}} + \frac{1-\lambda}{2} \right) \right)}{\kappa - \delta \left( \kappa - k_1 + \frac{1-\pi}{2} I_{SLO,2}^{B1=\emptyset} - (k_2 - k_1) \left( \lambda \pi I_{BMO,2}^{I_{B1=\emptyset}} + \frac{1-\lambda}{2} \right) \right)} \].

Case \( I_4 + I_5 \): \( \frac{\kappa - k_1}{\kappa} < \theta_2^I \leq \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}} \)

- **Informed traders**
  
  As \( \frac{\kappa - k_1}{\kappa} < \theta_2^I \), informed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever

  \[ \delta^{1-\lambda} (\kappa + k_1 - 1) > \kappa - k_1 \] and \[ \delta^{1-\lambda} (\kappa + k_1 - 1) \geq \theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 + \frac{1-\pi}{2} I_{SLO,2}^{B1=\emptyset} - (k_2 - k_1) \left( \frac{1-\lambda}{2} \right) \right) \].

  But since \( \frac{\kappa - k_1}{\kappa} < \theta_2^I = \theta_1^I \), it follows that the relevant inequality is the last one.

- **Uninformed traders**
  
  As \( \theta_2^I \leq \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}} \), uninformed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever

  \( (1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) > 0 \).

**Fifth step.** Based on the above, nobody at \( t = 1 \) has unilateral incentives to deviate from whenever

\[ (1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) > 0 \] and \[ \delta^{1-\lambda} (\kappa + k_1 - 1) \geq \theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 + \frac{1-\pi}{2} I_{SLO,2}^{B1=\emptyset} - (k_2 - k_1) \left( \frac{1-\lambda}{2} \right) \right) \].

These conditions are equivalent to

\[ PIN < \psi_{LO-NT}^U, \] and \[ \theta < \theta_1^I \leq \min \{ \overline{\theta}, \theta_{LO-DO} \}, \]

where \( \theta \) and \( \overline{\theta} \), are defined in (II.13) and

\[ \theta_{LO-DO} = \frac{\delta^{1-\lambda} (\kappa + k_1 - 1) - \delta \left( \kappa - k_1 + \frac{1-\pi}{2} I_{SLO,2}^{B1=\emptyset} - (k_2 - k_1) \left( \frac{1-\lambda}{2} \right) \right)}{\kappa - \delta \left( \kappa - k_1 + \frac{1-\pi}{2} I_{SLO,2}^{B1=\emptyset} - (k_2 - k_1) \left( \frac{1-\lambda}{2} \right) \right)} \]. (II.17)
Case $I_6: \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}} < \theta_2^I$

- **Informed traders**
  As $\frac{\kappa - k_1}{\kappa} < \theta_2^I$, informed traders at $t = 1$ have no incentives to deviate from the prescribed strategy profile whenever

  \[
  \delta \frac{1 - \lambda}{2} (\kappa + k_1 - 1) > \kappa - k_1 \quad \text{and} \quad \delta \frac{1 - \lambda}{2} (\kappa + k_1 - 1) \geq \theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 + \lambda \frac{1 - \pi_{SLO,2}}{2} I_{US, B_1 \neq \emptyset} - (k_2 - k_1) \frac{1 - \lambda}{2} \right).
  \]

  But, since $\frac{\kappa - k_1}{\kappa} < \theta_2^I = \theta_1^I$, it follows that the relevant inequality is the last one.

- **Uninformed traders**
  As $\frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}} < \theta_2^I$, uninformed traders at $t = 1$ have no incentives to deviate from the prescribed strategy profile whenever

  \[
  \delta \frac{1 - \lambda}{2} (k_1 - 1) > 0
  \]

  or, equivalently,

  \[
  k_1 > 1.
  \]

**Fifth step.** Based on the above, nobody at $t = 1$ has unilateral incentives to deviate from whenever

\[
\sigma < \kappa_{MO-LO}^I, \quad PIN < \psi_{LO-NT}^U, \quad \text{and} \quad \theta_1^I \leq \min \{ \theta, \hat{\theta}_{LO-DO} \},
\]

where defined in (II.13) and $\theta_{LO-DO}$ is defined in (II.17).

Therefore, the conditions under which nobody is willing to deviate at $t = 1$ are:

\[
\sigma < \kappa_{MO-LO}^I, \quad PIN < \psi_{LO-NT}^U, \quad \text{and} \quad \theta_1^I \leq \min \{ \theta, \hat{\theta}_{LO-DO} \},
\]

or

\[
PIN < \psi_{LO-NT}^U \quad \text{and} \quad \theta < \theta_1^I \leq \min \{ \theta, \hat{\theta}_{LO-DO} \},
\]

or

\[
k_1 > 1 \quad \text{and} \quad \theta_1^I \leq \theta_{LO-DO}.
\]

Finally, in the following tables we include the moves that are in the equilibrium path taking into account the conditions that must be satisfied if $(BLO, SLO, BLO, SLO)$ is the strategy profile chosen at $t = 1$ and the fact that in this case $\theta_2^I = \theta_1^I$.
Concerning uninformed traders, it follows that

<table>
<thead>
<tr>
<th>Condition</th>
<th>Optimal Choice of Uninformed Traders at $t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State of the Book</td>
</tr>
<tr>
<td>Case $U_1^{e_D}$</td>
<td>$Y^{3,D_K} \leq \frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Case $U_2^{e_D}$</td>
<td>$\frac{1}{2} &lt; Y^{3,D_K} \leq k_1$</td>
</tr>
<tr>
<td></td>
<td>or</td>
</tr>
<tr>
<td></td>
<td>$Y^{3,D_K} &gt; k_1$ and $\theta_{U_2}^\infty &gt; \theta_{Y^{3,D}}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Case $U_3^{e_D}$</td>
<td>$Y^{3,D_k_3} &gt; k_1$ and $\theta_{U_2}^\infty \leq \theta_{Y^{3,D}}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table II.23: Optimal choice of uninformed traders when the strategy profile at $t = 1$ is $(BLO, SLO, BLO, SLO)$. 
In relation to informed traders:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Optimal Choice of Informed Traders at ( t = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State of the Book</td>
</tr>
<tr>
<td>Case 1</td>
<td>( \theta^1 \leq \frac{\kappa - k_2}{\kappa - \frac{k_2 - k_1}{2}} )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 2</td>
<td>( \frac{\kappa - k_2}{\kappa - \frac{k_2 - k_1}{2}} &lt; \theta^1 \leq \frac{\kappa - k_1}{\kappa + \frac{k_2 - k_1}{2}} )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 3</td>
<td>( \frac{\kappa - k_1}{\kappa + \frac{k_2 - k_1}{2}} &lt; \theta^1 \leq \frac{\kappa - k_1}{\kappa - \frac{k_2 - k_1}{2}} )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 4</td>
<td>( \frac{\kappa - k_1}{\kappa - \frac{k_2 - k_1}{2}} &lt; \theta^1 \leq \frac{\kappa - k_1 + 1}{\kappa + \frac{k_2 - k_1}{2}} )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 5</td>
<td>( \frac{\kappa - k_1 + 1}{\kappa + \frac{k_2 - k_1}{2}} &lt; \theta^1 \leq \frac{\kappa - k_2}{\kappa - \frac{k_2 - k_1}{2}} )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table II.24: Optimal choice of informed traders when the strategy profile at \( t = 1 \) is \( (BLO, SLO, BLO, SLO) \).
Given the optimal response of traders at $t = 2$, find the optimal action for the traders at $t = 1$ in each of the 6 cases. However, given the nature of this particular equilibrium, we can group cases into the following and analyze them:

Case $I_1 + I_2 + I_3$: $\theta^I_2 \leq \frac{k - k_1}{\kappa}$
• Informed traders

As $\theta_2^I \leq \frac{\kappa-k_1}{\kappa}$, then informed traders at $t = 1$ have no incentives to deviate from the prescribed strategy profile whenever

$$\delta \frac{1-\lambda}{2} (\kappa + k_1 - 1) > \kappa - k_1 \quad \text{and} \quad \delta \frac{1-\lambda}{2} (\kappa + k_1 - 1) \geq \theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1-\lambda}{2} \right) \right),$$

since $I_{SLO,2}^{US, B_1 = \emptyset} = 0$ and $I_{BMO,2}^{IH, B_1 = \emptyset} = 1$.

• Uninformed traders

As $\theta_2^I \leq \frac{\kappa-k_1}{\kappa} \leq \frac{\kappa-k_1+1}{\kappa + \frac{1}{2}}$, uninformed traders at $t = 1$ have no incentives to deviate from the prescribed strategy profile whenever

$$(1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) \leq 0.$$  

Fifth step. From the previous two inequalities, nobody at $t = 1$ has unilateral incentives to deviate whenever

$$(1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) \leq 0$$

$$\delta \frac{1-\lambda}{2} (\kappa + k_1 - 1) > \kappa - k_1, \quad \text{and} \quad$$

$$\delta \frac{1-\lambda}{2} (\kappa + k_1 - 1) \geq \theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1-\lambda}{2} \right) \right).$$

This is equivalent to

$$PIN \geq \psi_{LO-NT}^U,$$

$$\sigma < \kappa_{MO-LO}^I \tau \quad \text{and} \quad \theta_1^I \leq \min \{ \underline{\theta}, \overline{\theta}_{LO-DO} \},$$

where $\underline{\theta}$ is defined in (II.13) and

$$\overline{\theta}_{LO-DO} \equiv \frac{\delta \frac{1-\lambda}{2} (\kappa + k_1 - 1) - \delta (\kappa - k_1 - (k_2 - k_1) (\lambda \pi + \frac{1-\lambda}{2}))}{\kappa - \delta (\kappa - k_1 - (k_2 - k_1) (\lambda \pi + \frac{1-\lambda}{2}))}. \quad \text{(II.19)}$$

Case $I_4 + I_5$: $\frac{\kappa-k_1}{\kappa} < \theta_2^I \leq \frac{\kappa-k_1+1}{\kappa + \frac{1}{2}}$

• Informed traders

As $\theta_2^I > \frac{\kappa-k_1}{\kappa}$, then informed traders at $t = 1$ have no incentives to deviate from the
prescribed strategy profile whenever
\[ \delta \frac{1-\lambda}{2} (\kappa + k_1 - 1) > \kappa - k_1 \] and
\[ \delta \frac{1-\lambda}{2} (\kappa + k_1 - 1) \geq \theta_1^R \kappa + (1 - \theta_1^R) \delta \left( \kappa - k_1 - (k_2 - k_1) \frac{1-\lambda}{2} \right), \]

since \( I_{SLO,2}^{US,B_1,=0} = 0 \). But, since \( \frac{\kappa - k_1}{\kappa} < \theta_2^R \theta_1^R = \theta_1^R \), the relevant inequality is the last one.

- **Uninformed traders**

As \( \theta_2^R \leq \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}} \), uninformed traders have no incentive to deviate if:
\[ (1-\lambda)(k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) \leq 0. \]

**Fifth step.** From the previous two restrictions, nobody at \( t = 1 \) has unilateral incentives to deviate whenever
\[ (1-\lambda)(k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) \leq 0 \]
and
\[ \delta \frac{1-\lambda}{2} (\kappa + k_1 - 1) \geq \theta_1^R \kappa + (1 - \theta_1^R) \delta \left( \kappa - k_1 - (k_2 - k_1) \frac{1-\lambda}{2} \right), \]
and this can be rewritten as
\[ \text{PIN} \geq \psi_{LO-NT}^U \text{ and } \theta < \theta_1^R \leq \min \{ \theta, \tilde{\theta}_{LO-DO} \}, \]

where \( \theta \) and \( \tilde{\theta} \) are defined in (II.13) and
\[ \tilde{\theta}_{LO-DO} = \frac{\delta \frac{1-\lambda}{2} (\kappa + k_1 - 1) - \delta (\kappa - k_1 - (k_2 - k_1) \frac{1-\lambda}{2})}{\kappa - \delta (\kappa - k_1 - (k_2 - k_1) \frac{1-\lambda}{2})}. \] (II.20)

**Case \( I_6 \):** \( \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}} < \theta_2^R \)

- **Informed traders**

As \( \theta_2^R > \frac{\kappa - k_1}{\kappa} \), then informed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever
\[ \delta \frac{1-\lambda}{2} (\kappa + k_1 - 1) > \kappa - k_1 \] and
\[ \delta \frac{1-\lambda}{2} (\kappa + k_1 - 1) \geq \theta_1^R \kappa + (1 - \theta_1^R) \delta \left( \kappa - k_1 - (k_2 - k_1) \frac{1-\lambda}{2} \right), \]

since \( I_{SLO,2}^{US,B_1,=0} = 0 \). But, since \( \frac{\kappa - k_1}{\kappa} < \theta_2^R = \theta_1^R \), the relevant inequality is the last one.
• Uninformed traders

As \( \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}} \) < \( \theta_2^I \), uninformed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever

\[
\delta \frac{1 - \lambda}{2} (k_1 - 1) \tau \leq 0 \iff k_1 - 1 \leq 0.
\]

This implies that in this case an equilibrium exists if \( k_1 = 1 \).

**Fifth step.** Hence, an equilibrium will have

\[
k_1 = 1 \text{ and } \delta \frac{1 - \lambda}{2} (\kappa + k_1 - 1) \geq \theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 - (k_2 - k_1) \frac{1 - \lambda}{2} \right).
\]

However, when \( k_1 = 1 \), \( \delta \frac{1 - \lambda}{2} \kappa < \kappa - 1 \), which implies that for an informed trader a MO provides higher expected profits than a LO. Hence, in this case we conclude that there is no equilibrium in which \((BLO, SLO, NT, NT)\) is the strategy profile chosen at \( t = 1 \).

Therefore, the conditions under which nobody is willing to deviate at \( t = 1 \) are:

\[
\sigma < \kappa^I_{MO-LO}, PIN \geq \psi^U_{LO-NT}, \text{ and } \theta_1^I \leq \min\{\theta_1, \theta_{LO-DO}\},
\]

or \( PIN \geq \psi^U_{LO-NT} \) and \( \theta < \theta_1^I \leq \min\{\theta_1, \theta_{LO-DO}\} \). (II.21)

Finally, in the following tables we include the moves that are in the equilibrium path taking into account the conditions that must be satisfied if \((BLO, SLO, NT, NT)\) is the strategy profile chosen at \( t = 1 \) and the fact that in this case \( \theta_2^U = \theta_1^I \) and \( \theta_2^I = \theta_1^U \).

Concerning the uninformed traders, we have

<table>
<thead>
<tr>
<th>Condition</th>
<th>Optimal Choice of Uninformed Traders at ( t = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>State of the Book</td>
<td>UB</td>
</tr>
<tr>
<td>Case ( U_1^{EP} ) ( \theta_2^U &gt; \theta_{Y4,i} )</td>
<td>((A_1^I, B_1^I))</td>
</tr>
<tr>
<td></td>
<td>((A_1^I, B_1^I))</td>
</tr>
<tr>
<td></td>
<td>((A_1^I, B_1^I + \tau))</td>
</tr>
<tr>
<td></td>
<td>((A_1^I, B_2^I))</td>
</tr>
<tr>
<td></td>
<td>((A_1^I - \tau, B_1^I))</td>
</tr>
<tr>
<td>Case ( U_2^{EP} ) ( \theta_2^U \leq \theta_{Y4,i} )</td>
<td>((A_1^I, B_1^I))</td>
</tr>
<tr>
<td></td>
<td>((A_1^I, B_1^I))</td>
</tr>
<tr>
<td></td>
<td>((A_1^I, B_1^I + \tau))</td>
</tr>
<tr>
<td></td>
<td>((A_1^I, B_2^I))</td>
</tr>
<tr>
<td></td>
<td>((A_1^I - \tau, B_1^I))</td>
</tr>
</tbody>
</table>

Table II.27: Optimal choice of uninformed traders at \( t = 2 \) when the strategy profile at \( t = 1 \) is \((BLO, SLO, NT, NT)\).
For informed traders, we have:

<table>
<thead>
<tr>
<th>Case</th>
<th>Condition</th>
<th>Optimal Choice of Informed Traders at $t=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1$</td>
<td>$\theta_1^I \leq \frac{\kappa - k_2}{\kappa - \frac{k_2 + k_1}{2}}$</td>
<td>(A$^1_1$, B$^1_1$) BMO SMO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^2_1$, B$^1_1$) BMO SMO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1$, B$^2_1$) BMO SMO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1$, B$^1_1 + \tau$) BMO SMO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1$, B$^1_1$) BMO SMO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1 - \tau$, B$^1_1$) BMO SMO</td>
</tr>
<tr>
<td>$I_2$</td>
<td>$\frac{\kappa - k_3}{\kappa - \frac{k_2 + k_1}{2}} &lt; \theta_1^I \leq \frac{\kappa - k_3}{\kappa + \frac{k_2 - k_1}{2}}$</td>
<td>(A$^1_1$, B$^1_1$) BMO SMO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^2_1$, B$^1_1$) BDO SMO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1$, B$^1_1 + \tau$) BMO SMO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1$, B$^2_1$) BDO SDO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1 - \tau$, B$^1_1$) BMO SMO</td>
</tr>
<tr>
<td>$I_3$</td>
<td>$\frac{\kappa - k_3}{\kappa + \frac{k_2 - k_1}{2}} &lt; \theta_1^I \leq \frac{\kappa - k_3}{\kappa}$</td>
<td>(A$^1_1$, B$^1_1$) BMO SMO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^2_1$, B$^1_1$) BDO SDO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1$, B$^1_1 + \tau$) BMO SMO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1$, B$^2_1$) BDO SDO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1 - \tau$, B$^1_1$) BMO SMO</td>
</tr>
<tr>
<td>$I_4$</td>
<td>$\frac{\kappa - k_3}{\kappa} &lt; \theta_1^I \leq \frac{\kappa - k_3}{\kappa + \frac{k_2 - k_1}{2}}$</td>
<td>(A$^1_1$, B$^1_1$) BDO SDO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^2_1$, B$^1_1$) BDO SDO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1$, B$^1_1 + \tau$) BMO SMO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1$, B$^2_1$) BDO SDO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1 - \tau$, B$^1_1$) BMO SMO</td>
</tr>
<tr>
<td>$I_5$</td>
<td>$\frac{\kappa - k_3}{\kappa + \frac{k_2 - k_1}{2}} &lt; \theta_1^I \leq \frac{\kappa - k_3 + 1}{\kappa + \frac{k_2 - k_1}{2}}$</td>
<td>(A$^1_1$, B$^1_1$) BDO SDO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^2_1$, B$^1_1$) BDO SDO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1$, B$^1_1 + \tau$) BDO SDO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1$, B$^2_1$) BDO SDO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1 - \tau$, B$^1_1$) BMO SDO</td>
</tr>
<tr>
<td>$I_6$</td>
<td>$\frac{\kappa - k_3 + 1}{\kappa + \frac{k_2 - k_1}{2}} &lt; \theta_1^I$</td>
<td>(A$^1_1$, B$^1_1$) BDO SDO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^2_1$, B$^1_1$) BDO SDO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1$, B$^1_1 + \tau$) BDO SDO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1$, B$^2_1$) BDO SDO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(A$^1_1 - \tau$, B$^1_1$) BDO SDO</td>
</tr>
</tbody>
</table>

Table II.28: Optimal choice of informed traders at $t = 2$ when the strategy profile at $t = 1$ is (BLO, SLO, NT, NT).
\( \mathcal{E}_D^5 : (BDO, SDO, BLO, SLO) \)

**First step.** In this case \( \Omega_0 = 0 \), \( \Omega_1 = 0 \), \( \Omega_2 = 0 \), \( \Omega_3 = 1 \), \( \Gamma_0 = 0 \), \( \Gamma_1 = 0 \), \( \Gamma_2 = 1 \), and \( \Gamma_3 = 0 \). Moreover, \( \theta_1^\ell \leq \theta_1^u \) and \( \theta_2^\ell \leq \theta_2^u \).

**Second step.** Using Bayes’ rule we obtain \( X_5^5.D = 0 \), \( Y_5^5.D = 0 \) and \( Z_5^5.D = 1 \).

**Third step.** Using step 2, at \( t = 2 \) the expected payoffs of an uninformed buyer are as follows:

### Table II.29: Expected profits of uninformed buyers at \( t = 2 \) when the strategy profile at \( t = 1 \) is \( (BDO, SDO, BLO, SLO) \).

<table>
<thead>
<tr>
<th>State of the Book</th>
<th>UB</th>
<th>BMO</th>
<th>BDO</th>
<th>BLO</th>
<th>NT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (A_1^1, B_1^1) )</td>
<td>(-k_1 \tau)</td>
<td>0</td>
<td>( p_{BLO}^{UB} )</td>
<td>( k_1 - Z_5^5.D \kappa - 1 ) ( \tau )</td>
<td>0</td>
</tr>
<tr>
<td>( (A_1^2, B_1^1) )</td>
<td>(-k_2 \tau)</td>
<td>(-\theta_2^\ell \kappa - k_2 \kappa \tau)</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( (A_1^1, B_1^1 + \tau) )</td>
<td>(-k_1 \tau)</td>
<td>(-\theta_2^\ell \frac{k_2}{2} \tau)</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( (A_1^2, B_1^2) )</td>
<td>(-k_1 \tau)</td>
<td>( \theta_2^u \frac{k_2}{2} - k_2 \tau)</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( (A_1^1 - \tau, B_1^1) )</td>
<td>(- (k_1 - 1) \tau)</td>
<td>( \theta_2^u \frac{k_2}{2} \tau)</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

### Table II.30: Optimal strategies of uninformed traders at \( t = 2 \) when the strategy profile at \( t = 1 \) is \( (BDO, SDO, BLO, SLO) \).

By symmetry, we can find the expected profits of uninformed sellers. Hence, the optimal strategies for the uninformed traders are:

<table>
<thead>
<tr>
<th>State of the Book</th>
<th>UB</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (A_1^1, B_1^1) )</td>
<td>NT</td>
<td>NT</td>
</tr>
<tr>
<td>( (A_2^1, B_1^1) )</td>
<td>NT</td>
<td>SDO</td>
</tr>
<tr>
<td>( (A_1^1, B_1^1 + \tau) )</td>
<td>NT</td>
<td>SDO</td>
</tr>
<tr>
<td>( (A_1^2, B_1^2) )</td>
<td>BDO</td>
<td>NT</td>
</tr>
<tr>
<td>( (A_1^1 - \tau, B_1^1) )</td>
<td>BDO</td>
<td>NT</td>
</tr>
</tbody>
</table>

### Table II.31: Expected profits of informed buyers at \( t = 2 \) when the strategy profile at \( t = 1 \) is \( (BDO, SDO, BLO, SLO) \).

For the informed buyer, using step 2, the expected profits are

### Table II.31: Expected profits of informed buyers at \( t = 2 \) when the strategy profile at \( t = 1 \) is \( (BDO, SDO, BLO, SLO) \).

\[
p_{BLO,2}^{IH}(B_1 = \emptyset) = \frac{(1 - \theta_1^u)(1 - \Gamma_3)}{\frac{1}{\kappa + 1} + \frac{1}{\kappa + 1}} = 0 \text{ because } \Gamma_3 = 0 \text{ and } \Omega_3 = 1. \text{ By symmetry, we can}
\]
find the expected profits of informed sellers. Hence, it follows that in this case the optimal strategies of informed traders at $t = 2$ are given in Table II.18.

**Fourth step.** Given the optimal response of traders at $t = 2$, find the optimal action for the traders at $t = 1$ in each of the 6 cases. However, given the nature of this particular equilibrium, we can group cases into the following and analyze them:

<table>
<thead>
<tr>
<th>Case</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1 + I_2 + I_3 : \theta_2^I \leq \frac{\kappa - k_1}{\kappa}$</td>
<td></td>
</tr>
</tbody>
</table>

- **Informed traders**

  As $\theta_2^I \leq \frac{\kappa - k_1}{\kappa}$, then informed traders at $t = 1$ have no incentives to deviate from the prescribed strategy profile whenever

  $$
  \theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1 - \lambda}{2} \right) \right) > \kappa - k_1 
  $$

- **Uninformed traders**

  As $\theta_2^I \leq \frac{\kappa - k_1}{\kappa} \leq \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}}$, uninformed traders at $t = 1$ have no incentives to deviate from the prescribed strategy profile whenever

  $$
  (1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) > 0.
  $$

**Fifth step.** From the previous two inequalities, nobody at $t = 1$ has unilateral incentives to deviate from ($BDO, SDO, BLO, SLO$) whenever

$$
\theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1 - \lambda}{2} \right) \right) > \kappa - k_1,
$$

$$
\theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1 - \lambda}{2} \right) \right) > \delta \frac{1 - \lambda}{2} (\kappa + k_1 - 1), 	ext{ and}
$$

$$
(1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) > 0.
$$

From the previous inequalities, nobody at $t = 1$ has unilateral incentives to deviate from ($BDO, SDO, BLO, SLO$) whenever

$$
\theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1 - \lambda}{2} \right) \right) > \kappa - k_1,
$$

$$
\theta_1^I \kappa + (1 - \theta_1^I) \delta \left( \kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1 - \lambda}{2} \right) \right) > \delta \frac{1 - \lambda}{2} (\kappa + k_1 - 1), 	ext{ and}
$$

$$
(1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) > 0
$$
or, equivalently,
\[
PIN < \psi_{LO-NT}^U, \\
\theta_1^l > \max\{\theta_{MO-DO}, \theta_{LO-DO}\}, \\
\theta_2^l \leq \theta \text{ and } \theta_2^l \leq \theta_1^l,
\]
where \(\theta, \theta_{MO-DO}, \theta_{LO-DO}\) are defined in (II.13), (II.15) and (II.19), respectively.

Case I4 + I5: \(\frac{\kappa-k_1}{\kappa} < \theta_2^l \leq \frac{\kappa-k_1+1}{\kappa+\frac{1}{2}}\)

• Informed traders

As \(\theta_2^l > \frac{\kappa-k_1}{\kappa}\), then informed traders at \(t=1\) have no incentives to deviate from the prescribed strategy profile whenever
\[
\theta_1^l \kappa + (1-\theta_1^l) \delta \left( \kappa - k_1 - (k_2 - k_1) \frac{1-\lambda}{2} \right) > \kappa - k_1 \text{ and}
\]
\[
\theta_1^l \kappa + (1-\theta_1^l) \delta \left( \kappa - k_1 - (k_2 - k_1) \frac{1-\lambda}{2} \right) > \delta \frac{1-\lambda}{2} (\kappa + k_1 - 1).
\]

since \(I_{US,B_1=\emptyset}=0\). But, since \(\frac{\kappa-k_1}{\kappa} < \theta_2^l \leq \theta_1^l\), the relevant inequality is the last one.

• Uninformed traders

As \(\theta_2^l \leq \frac{\kappa-k_1+1}{\kappa+\frac{1}{2}}\), uninformed traders at \(t=1\) have no incentives to deviate from the prescribed strategy profile whenever
\[
(1-\lambda) (k_1-1) - \lambda \pi (\kappa - (k_1 - 1)) > 0.
\]

**Fifth step.** From the previous two inequalities, nobody at \(t=1\) has unilateral incentives to deviate from \((BDO, SDO, BLO, SLO)\) whenever
\[
\theta_1^l \kappa + (1-\theta_1^l) \delta \left( \kappa - k_1 - (k_2 - k_1) \frac{1-\lambda}{2} \right) > \delta \frac{1-\lambda}{2} (\kappa + k_1 - 1) \text{ and}
\]
\[
(1-\lambda) (k_1-1) - \lambda \pi (\kappa - (k_1 - 1)) > 0.
\]

These conditions are equivalent to
\[
PIN < \psi_{LO-NT}^U, \\
\tilde{\theta}_{LO-DO} < \theta_1^l, \tilde{\theta} < \theta_2^l \leq \tilde{\theta}, \text{ and } \theta_2^l \leq \theta_1^l,
\]
where \(\theta, \tilde{\theta}, \tilde{\theta}_{LO-DO}\) are defined in (II.13) and (II.20), respectively.

Case I6: \(\frac{\kappa-k_1+1}{\kappa+\frac{1}{2}} < \theta_2^l\)

• Informed traders
The same conditions apply for the informed as in case $I_4$ and $I_5$. Hence, informed traders at $t = 1$ have no incentives to deviate from the prescribed strategy profile whenever

$$\theta^I_1\kappa + (1 - \theta^I_1)\delta \left(\kappa - k_1 - (k_2 - k_1)\frac{1 - \lambda}{2}\right) > \frac{1 - \lambda}{2}(\kappa + k_1 - 1).$$

- **Uninformed traders**

As $\frac{\kappa - k_1 + 1}{\kappa + 2} < \theta^I_2$, uninformed traders at $t = 1$ have no incentives to deviate from the prescribed strategy profile whenever

$$\delta \frac{1 - \lambda}{2}(k_1 - 1)\tau > 0 \iff k_1 - 1 > 0.$$

**Fifth step.** From the previous two inequalities, nobody at $t = 1$ has unilateral incentives to deviate from $(BDO, SDO, BLO, SLO)$ whenever

$$\theta^I_1\kappa + (1 - \theta^I_1)\delta \left(\kappa - k_1 - (k_2 - k_1)\frac{1 - \lambda}{2}\right) > \frac{1 - \lambda}{2}(\kappa + k_1 - 1) \text{ and } k_1 > 1.$$

These conditions are equivalent to

$$k_1 > 1, \tilde{\theta}_{LO-DO} < \theta^I_1 \text{ and } \overline{\theta} < \theta^I_2 \leq \theta^I_1,$$

where $\tilde{\theta}_{LO-DO}$ is defined in (II.20).

Therefore, nobody at $t = 1$ has unilateral incentives to deviate from $(BDO, SDO, BLO, SLO)$ whenever

$$PIN < \psi^I_{LO-NT}, \theta^I_1 > \max\{\overline{\theta}_{MO-DO}, \tilde{\theta}_{LO-DO}\}, \text{ and } \theta^I_2 \leq \min\{\overline{\theta}, \theta^I_1\},$$

or $PIN < \psi^I_{LO-NT}, \theta^I_{LO-DO} < \theta^I_1, \text{ and } \overline{\theta} < \theta^I_2 \leq \min\{\overline{\theta}, \theta^I_1\},$ \hspace{1cm} (II.22)

or $k_1 > 1, \tilde{\theta}_{LO-DO} < \theta^I_1, \text{ and } \overline{\theta} < \theta^I_2 \leq \theta^I_1.$

Finally, we consider only the moves that are in the equilibrium path taking into account the conditions that must be satisfied if $(BDO, SDO, BLO, SLO)$ is the strategy profile chosen at $t = 1$. For uninformed traders,

<table>
<thead>
<tr>
<th>State of the Book</th>
<th>UB</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(A_1^I, B_1^I)$</td>
<td>$NT$</td>
<td>$NT$</td>
</tr>
<tr>
<td>$(A_2^I, B_1^I)$</td>
<td>$NT$</td>
<td>$SDO$</td>
</tr>
<tr>
<td>$(A_1^I, B_1^I + \tau)$</td>
<td>$NT$</td>
<td>$SDO$</td>
</tr>
<tr>
<td>$(A_1^I, B_1^I)$</td>
<td>$BDO$</td>
<td>$NT$</td>
</tr>
<tr>
<td>$(A_1^I - \tau, B_1^I)$</td>
<td>$BDO$</td>
<td>$NT$</td>
</tr>
</tbody>
</table>

Table II.32: Optimal choice of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BDO, SDO, BLO, SLO)$.
For informed traders,

<table>
<thead>
<tr>
<th>Condition</th>
<th>Optimal Choice of Informed Traders at $t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State of the Book</td>
</tr>
<tr>
<td>Case $I_1$</td>
<td>$\theta^I_2 \leq \frac{\kappa - k_2}{\kappa - \frac{k_2}{2}}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Case $I_2$</td>
<td>$\frac{\kappa - k_2}{\kappa - \frac{k_2}{2}} &lt; \theta^I_2 \leq \frac{\kappa - k_1}{\kappa + \frac{k_2}{2}}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Case $I_3$</td>
<td>$\frac{\kappa - k_1}{\kappa + \frac{k_2}{2}} &lt; \theta^I_2 \leq \frac{\kappa - k_1}{\kappa - \frac{k_2}{2}}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Case $I_4$</td>
<td>$\frac{\kappa - k_1}{\kappa - \frac{k_2}{2}} &lt; \theta^I_2 \leq \frac{\kappa - k_1}{\kappa + \frac{k_2}{2}}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Case $I_5$</td>
<td>$\frac{\kappa - k_1}{\kappa - \frac{k_2}{2}} &lt; \theta^I_2 \leq \frac{\kappa - k_1 + 1}{\kappa + \frac{k_2}{2}}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Case $I_6$</td>
<td>$\frac{\kappa - k_1 + 1}{\kappa + \frac{k_2}{2}} &lt; \theta^I_2$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table II.33: Optimal choice of informed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BDO, SDO, BLO, SLO)$. 

52
\( \mathcal{E}^D_6 : (BDO, SDO, NT, NT) \)

**First step.** In this case \( \Omega_0 = 0, \Omega_1 = 0, \Omega_2 = 0, \Omega_3 = 1, \Gamma_0 = 1, \Gamma_1 = 0, \Gamma_2 = 0, \) and \( \Gamma_3 = 0. \) Moreover, \( \theta^U_2 \leq \theta^U_1 \) and \( \theta^D_2 \leq \theta^D_1. \)

**Second step.** Using Bayes’ rule,

\[
X^{6,D} = 0, \quad Y^{6,D} = y \in [0,1] \text{ and } Z^{6,D} = 1, \\
p_{BLO,2}^{UB,B_1=\emptyset} = p_{SLO,2}^{US,B_1=\emptyset} = (1 - \theta^U_1)^\frac{\pi}{2}, \text{ and } p_{BLO,2}^{IH,B_1=\emptyset} = p_{SLO,2}^{IL,B_1=\emptyset} = 0
\]

**Third step.** Using steps 1 and 2, the expected profits of uninformed traders at \( t = 2 \) are given by

<table>
<thead>
<tr>
<th>( UB )</th>
<th>( BMO )</th>
<th>( BDO )</th>
<th>( BLO )</th>
<th>( NT )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (A^1_1, B^1_1) )</td>
<td>(-k_1 \tau)</td>
<td>(-k_2 \tau)</td>
<td>(- (1 - \theta^U_1)^\frac{\pi}{2} (\kappa - (k_1 + 1)) \tau)</td>
<td>(0)</td>
</tr>
<tr>
<td>( (A^2_1, B^1_1) )</td>
<td>(-k_2 \tau)</td>
<td>(- \frac{\theta^U_2}{2} (k_2 - k_1) \tau)</td>
<td>(0)</td>
<td>(0)</td>
</tr>
<tr>
<td>( (A^1_1, B^1_1 + \tau) )</td>
<td>( (Y^6,D_K - k_1) \tau)</td>
<td>( \theta^U_2 (Y^6,D_K - \frac{1}{2}) \tau)</td>
<td>(0)</td>
<td>(0)</td>
</tr>
<tr>
<td>( (A^1_1, B^1_1 - \tau) )</td>
<td>(-k_1 \tau)</td>
<td>(- \frac{\theta^D_2}{2} (Y^6,D_K - \frac{1}{2}) \tau)</td>
<td>(0)</td>
<td>(0)</td>
</tr>
</tbody>
</table>

Table II.34: Expected profits of uninformed buyers at \( t = 2 \) when the strategy profile at \( t = 1 \) is \( (BDO, SDO, NT, NT) \).

By symmetry, we can find the expected profits of uninformed sellers. Hence, the optimal strategies for the uninformed are:

<table>
<thead>
<tr>
<th>State of the Book</th>
<th>( UB )</th>
<th>( US )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (A^1_1, B^1_1) )</td>
<td>( NT )</td>
<td>( NT )</td>
</tr>
<tr>
<td>( (A^2_1, B^1_1) )</td>
<td>( NT )</td>
<td>( SDO )</td>
</tr>
<tr>
<td>( (A^1_1, B^1_1 + \tau) )</td>
<td>( BDO ) if ( Y^6,D_K \leq \frac{1}{2} ) &amp; ( \theta^D_2 &gt; \theta_{Y^6,D} ) &amp; ( Y^6,D_K \geq k_1 )</td>
<td>( NT ) if ( Y^6,D_K \geq \frac{1}{2} ) &amp; ( SDO ) if ( Y^6,D_K &lt; \frac{1}{2} )</td>
</tr>
<tr>
<td>( (A^1_1, B^1_1 - \tau) )</td>
<td>( BDO ) if ( Y^6,D_K \leq \frac{1}{2} ) | ( SMO ) if ( Y^6,D_K \geq 1 ) &amp; ( \theta \leq \theta_{Y^6,D} )</td>
<td></td>
</tr>
<tr>
<td>( (A^1_1, B^2_1) )</td>
<td>( BDO )</td>
<td>( NT )</td>
</tr>
</tbody>
</table>

Table II.35: Optimal strategies of uninformed traders at \( t = 2 \) when the strategy profile at \( t = 1 \) is \( (BDO, SDO, NT, NT) \).
Concerning the informed traders, using the fact that \( p_{BLO,2}^{IH}(O_1 = \emptyset) = 0 \), at \( t = 2 \) their expected profits are as follows:

<table>
<thead>
<tr>
<th>( IH )</th>
<th>( BMO )</th>
<th>( BDO )</th>
<th>( BLO )</th>
<th>( NT )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (A_1^1, B_1^1) )</td>
<td>( (\kappa - k_1) \tau )</td>
<td>( \theta_2^I \kappa \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( (A_2^2, B_1^1) )</td>
<td>( (\kappa - k_2) \tau )</td>
<td>( \theta_2^I \left( \kappa - \frac{k_2 - k_1}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( (A_1^1, B_1^1 + \tau) )</td>
<td>( (\kappa - k_1) \tau )</td>
<td>( \theta_2^I \left( \kappa - \frac{k_1}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( (A_1^1, B_1^1) )</td>
<td>( (\kappa - k_1) \tau )</td>
<td>( \theta_2^I \left( \kappa + \frac{k_2 - k_1}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( (A_1^1 - \tau, B_1^1) )</td>
<td>( (\kappa - k_1 + 1) \tau )</td>
<td>( \theta_2^I \left( \kappa + \frac{k_1}{2} \right) \tau )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table II.36: Expected profits of informed buyers at \( t = 2 \) when the strategy profile at \( t = 1 \) is \( (BDO, SDO, NT, NT) \).

By symmetry, we can find the expected profits of informed sellers. Concerning informed traders, the optimal strategies for these traders at \( t = 2 \) are given in Table II.18.

**Fourth step.** Given the optimal response of traders at \( t = 2 \), find the optimal action for the traders at \( t = 1 \) in each of the 6 cases. However, given the nature of this particular equilibrium, we can group cases into the following and analyze them:

**Case** \( I_1 + I_2 + I_3 : \theta_2^I \leq \frac{\kappa - k_1}{\kappa} \)

- **Informed traders**

As \( \theta_2^I \leq \frac{\kappa - k_1}{\kappa} \), then informed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever

\[
\theta_2^I \kappa + (1 - \theta_2^I) \delta \left( \kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1 - \lambda}{2} \right) \right) > \kappa - k_1 \text{ and }
\theta_2^I \kappa + (1 - \theta_2^I) \delta \left( \kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1 - \lambda}{2} \right) \right) > \frac{\delta (1 - \lambda)}{2} (\kappa + k_1 - 1),
\]

since \( I_{SLO,2}^{US,B_1=\emptyset} = 0 \) and \( I_{BMO,2}^{IH,B_1=\emptyset} = 1 \).

- **Uninformed traders**

As \( \theta_2^I \leq \frac{\kappa - k_1}{\kappa} \leq \frac{\kappa - k_1 + 1}{\kappa + \frac{3}{2}} \), uninformed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever

\[
(1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) \leq 0.
\]

**Fifth step.** From the previous two inequalities, nobody at \( t = 1 \) has unilateral incentives to
deviate from \((BDO, SDO, NT, NT)\) whenever

\[
(1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) \leq 0,
\]

\[
\theta^I_1 \kappa + (1 - \theta^I_1) \delta \left( \kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1 - \lambda}{2} \right) \right) > \kappa - k_1 \quad \text{and}
\]

\[
\theta^I_1 \kappa + (1 - \theta^I_1) \delta \left( \kappa - k_1 - (k_2 - k_1) \left( \lambda \pi + \frac{1 - \lambda}{2} \right) \right) > \frac{\delta (1 - \lambda)}{2} (\kappa + k_1 - 1),
\]

Therefore, nobody at \(t = 1\) has unilateral incentives to deviate from \((BDO, SDO, NT, NT)\) whenever

\[
PIN \geq \psi^I_{LO-NT},
\]

\[
\theta^I_1 > \max\{\overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO}\} \quad \text{and}
\]

\[
\theta^I_2 \leq \theta,
\]

where \(\theta, \overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO}\) are defined in \((II.13)\), \((II.15)\), and \((II.19)\) respectively.

Case \(I_4 + I_5: \frac{\kappa - k_1}{\kappa} < \theta^I_2 \leq \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}}\)

- **Informed traders**

  As \(\theta^I_2 > \frac{\kappa - k_1}{\kappa}\), then informed traders at \(t = 1\) have no incentives to deviate from the prescribed strategy profile whenever

  \[
  \theta^I_1 \kappa + (1 - \theta^I_1) \delta \left( \kappa - k_1 - (k_2 - k_1) \frac{1 - \lambda}{2} \right) > \kappa - k_1 \quad \text{and}
  \]

  \[
  \theta^I_1 \kappa + (1 - \theta^I_1) \delta \left( \kappa - k_1 - (k_2 - k_1) \frac{1 - \lambda}{2} \right) > \frac{\delta (1 - \lambda)}{2} (\kappa + k_1 - 1).
  \]

  since \(I^U_{SLO_2} = \emptyset\). But, since \(\frac{\kappa - k_1}{\kappa} < \theta^I_2 \leq \theta^I_1\), the relevant inequality is the last one.

- **Uninformed traders**

  As \(\theta^I_2 \leq \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}}\), uninformed traders at \(t = 1\) have no incentives to deviate from the prescribed strategy profile whenever

  \[
  (1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) \leq 0.
  \]

**Fifth step.** From the previous two inequalities, nobody at \(t = 1\) has unilateral incentives to deviate from \((BDO, SDO, NT, NT)\) whenever

\[
(1 - \lambda) (k_1 - 1) - \lambda \pi (\kappa - (k_1 - 1)) \leq 0 \quad \text{and}
\]

\[
\theta^I_1 \kappa + (1 - \theta^I_1) \delta \left( \kappa - k_1 - (k_2 - k_1) \frac{1 - \lambda}{2} \right) > \frac{(1 - \lambda)}{2} \delta (\kappa + k_1 - 1).
\]
These conditions are equivalent to
\[ PIN \geq \psi_{LO-NT}, \]
\[ \theta < \theta'_2 \leq \bar{\theta}, \]
\[ \tilde{\theta}_{LO-DO} < \theta'_1 \text{ and } \theta'_2 \leq \theta'_1, \]
where \( \theta, \bar{\theta}, \tilde{\theta}_{LO-DO} \) are defined in (II.13) and (II.20) respectively.

Case I_6: \( \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}} < \theta'_2 \)

- **Informed traders.**

  The same conditions apply for the informed as in case I_4 and I_5. Hence, informed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever

  \[ \theta'_1 \kappa + (1 - \theta'_1) \delta \left( \kappa - k_1 - (k_2 - k_1) \frac{1 - \lambda}{2} \right) > \frac{1 - \lambda}{2} \delta (\kappa + k_1 - 1). \]

- **Uninformed traders**

  As \( \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}} < \theta'_2 \), uninformed traders at \( t = 1 \) have no incentives to deviate from the prescribed strategy profile whenever

  \[ \delta \frac{1 - \lambda}{2} (k_1 - 1) \tau \leq 0 \iff k_1 - 1 \leq 0. \]

This implies that in this case an equilibrium exists if \( k_1 = 1 \).

**Fifth step.** From the previous two inequalities, nobody at \( t = 1 \) has unilateral incentives to deviate from \((BDO, SDO, NT, NT)\) whenever

\[ \theta'_1 \kappa + (1 - \theta'_1) \delta \left( \kappa - k_1 - (k_2 - k_1) \frac{1 - \lambda}{2} \right) > \frac{1 - \lambda}{2} \delta (\kappa + k_1 - 1) \text{ and } k_1 = 1. \]

These conditions are equivalent to

\[ \tilde{\theta}_{LO-DO} < \theta'_1, \quad \bar{\theta} < \theta'_2 \leq \theta'_1 \text{ and } k_1 = 1, \]

where \( \bar{\theta} \) and \( \tilde{\theta}_{LO-DO} \) are defined in (II.13) and (II.20) respectively.

Therefore, nobody at \( t = 1 \) has unilateral incentives to deviate from \((BDO, SDO, NT, NT)\) whenever

\[ PIN \geq \psi^U_{LO-NT}, \theta'_1 > \max\{\overline{\theta}_{MO-DO}, \tilde{\theta}_{LO-DO}\}, \text{ and } \theta'_2 \leq \overline{\theta}, \]

or \[ PIN \geq \psi^U_{LO-NT}, \tilde{\theta}_{LO-DO} < \theta'_1, \text{ and } \overline{\theta} < \theta'_2 \leq \overline{\theta}, \]

or \[ \tilde{\theta}_{LO-DO} < \theta'_1, \overline{\theta} < \theta'_2 \leq \theta'_1 \text{ and } k_1 = 1. \]
Finally, in the following tables we include the moves that are in the equilibrium path taking into account the conditions that must be satisfied if \((BDO, SDO, NT, NT)\) is the strategy profile chosen at \(t = 1\).

For uninformed traders,

<table>
<thead>
<tr>
<th>State of the Book</th>
<th>UB</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>((A_1^1, B_1^1))</td>
<td>NT</td>
<td>NT</td>
</tr>
<tr>
<td>((A_2^1, B_1^1))</td>
<td>NT</td>
<td>SDO</td>
</tr>
<tr>
<td>((A_1^1, B_1^1))</td>
<td>BDO</td>
<td>NT</td>
</tr>
</tbody>
</table>

Table II.37: Optimal choice of uninformed traders at \(t = 2\) when the strategy profile at \(t = 1\) is \((BDO, SDO, NT, NT)\).

For informed traders,

<table>
<thead>
<tr>
<th>Condition</th>
<th>Optimal Choice of Informed Traders at (t = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>State of the Book</td>
</tr>
<tr>
<td>Case (I_1) (\theta_2^I \leq \frac{\kappa-k_3}{\kappa+\frac{k_2-k_3}{2}})</td>
<td>((A_1^1, B_1^1))</td>
</tr>
<tr>
<td></td>
<td>((A_2^1, B_1^1))</td>
</tr>
<tr>
<td></td>
<td>((A_1^1, B_2^1))</td>
</tr>
<tr>
<td>Case (I_2) (\frac{\kappa-k_3}{\kappa+\frac{k_2-k_3}{2}} &lt; \theta_2^I \leq \frac{\kappa-k_3}{\kappa+\frac{k_2-k_3}{2}})</td>
<td>((A_1^1, B_1^1))</td>
</tr>
<tr>
<td></td>
<td>((A_2^1, B_1^1))</td>
</tr>
<tr>
<td></td>
<td>((A_1^1, B_2^1))</td>
</tr>
<tr>
<td>Case (I_3) (\frac{\kappa-k_3}{\kappa} &lt; \theta_2^I \leq \frac{\kappa-k_3}{\kappa+\frac{k_2-k_3}{2}})</td>
<td>((A_1^1, B_1^1))</td>
</tr>
<tr>
<td></td>
<td>((A_2^1, B_1^1))</td>
</tr>
<tr>
<td></td>
<td>((A_1^1, B_2^1))</td>
</tr>
<tr>
<td>Case (I_4) (\frac{\kappa-k_3}{\kappa+\frac{k_2-k_3}{2}} &lt; \theta_2^I \leq \frac{\kappa-k_3+1}{\kappa+\frac{k_2-k_3}{2}})</td>
<td>((A_1^1, B_1^1))</td>
</tr>
<tr>
<td></td>
<td>((A_2^1, B_1^1))</td>
</tr>
<tr>
<td></td>
<td>((A_1^1, B_2^1))</td>
</tr>
<tr>
<td>Case (I_5) (\frac{\kappa-k_3+1}{\kappa+\frac{k_2-k_3}{2}} &lt; \theta_2^I \leq \frac{\kappa-k_3+1}{\kappa+\frac{k_2-k_3}{2}})</td>
<td>((A_1^1, B_1^1))</td>
</tr>
<tr>
<td></td>
<td>((A_2^1, B_1^1))</td>
</tr>
<tr>
<td></td>
<td>((A_1^1, B_2^1))</td>
</tr>
</tbody>
</table>

Table II.38: Optimal choice of informed traders at \(t = 2\) when the strategy profile at \(t = 1\) is \((BDO, SDO, NT, NT)\).
Finally, note that substituting $k_1 = 1$ into the expressions of $\kappa_{MO-LO}$ and $\psi_{LO-NT}$, we have that
\[
\kappa_{MO-LO}^I = \frac{1}{1 - \frac{1}{2} \delta (1 - \lambda)}, \text{ and } \psi_{LO-NT}^U = 0.
\]
Moreover, since $\kappa_{MO-LO}^I < 2$, it follows that $\kappa_{MO-LO}^I < \sigma$ and $PIN \geq \psi_{LO-NT}^U$. Therefore, using (II.14)-(II.23), we have that when $k_1 = 1$, the conditions related to $E_1^D$, $E_3^D$ and $E_5^D$ do not hold. Moreover, when $k_1 = 1$, $\delta \frac{1-\lambda}{2} \kappa < \kappa - 1$, which implies that an informed trader at $t = 1$ prefers a $MO$ to a $LO$. Hence, $E_1^D$ is not feasible when $k_1 = 1$. Therefore, in this case we have that $\theta^I_1 \leq \theta_{MO-DO}$ and $(BDO, SDO, NT, NT)$ is the optimal strategy profile at $t = 1$ if
\[
\theta^I_1 > \max\{\theta_{MO-DO}, \theta_{LO-DO}\} \text{ and } \theta^I_2 \leq \min\{\theta, \theta^I_1\},
\]
or $\theta_{LO-DO} < \theta^I_1$ and $\theta < \theta^I_2 \leq \theta^I_1$.

**Proof of Proposition 2.**

**Case A** We consider the same four possible cases depending on the initial conditions in the single-venue market.

**Case A.1:** $\sigma < \kappa_{MO-LO}^I$ and $PIN < \psi_{LO-NT}^U$.

In this case, we start with a market in which the equilibrium is $E_3^{ND}$, where conditions (II.6) and (II.7) are satisfied. In this case, when we add the $DP$ out of the 6 equilibria, there are only two possible equilibria that satisfy these conditions: $E_3^D$ and $E_5^D$. From Lemma C.2 we can see that $E_3^D$ is an equilibrium if conditions (II.18) are satisfied. This can be rewritten as
\[
\theta^I_1 \leq \min\{\theta, \theta_{LO-DO}\}
\]
or $\theta < \theta^I_1 \leq \theta_{LO-DO}$.

Using the relationship between cutoffs (C.4) (in the paper), we know that $\theta_{LO-DO} \leq \theta^I_{LO-DO}$. Then, we consider the following cases: I) $\theta < \theta_{LO-DO}$, II) $\theta_{LO-DO} \leq \theta < \theta^I_{LO-DO}$, and III) $\theta^I_{LO-DO} \leq \theta$.

**Case I:** $\theta < \theta_{LO-DO}$. As $\theta_{LO-DO} \leq \theta^I_{LO-DO}$, it follows that $\theta < \theta^I_{LO-DO}$. Hence, the conditions that guarantee that $E_3^D$ is an equilibrium can be rewritten as
\[
\theta^I_1 \leq \theta
\]
or $\theta < \theta^I_1 \leq \theta_{LO-DO}$, which can be further simplified as
\[
\theta^I_1 \leq \theta_{LO-DO}.
\]

**Case II:** $\theta_{LO-DO} \leq \theta < \theta^I_{LO-DO}$. In this case, the conditions that guarantee that $E_3^D$ is an
equilibrium can be rewritten as

$$\theta_1^I \leq \theta_{LO-DO}.$$  

**Case III:** \(\hat{\theta}_{LO-DO} \leq \theta.\) In this case, the conditions that guarantee that \(E^D_3\) is an equilibrium can be rewritten as

$$\theta_1^I \leq \hat{\theta}_{LO-DO}.$$  

Consequently, we conclude that \(E^D_3\) is an equilibrium whenever

$$\theta_1^I \leq \theta_{LO-DO} \quad \text{if} \quad \theta < \theta_{LO-DO},$$

or

$$\theta_1^I \leq \theta \quad \text{if} \quad \theta_{LO-DO} \leq \theta < \hat{\theta}_{LO-DO},$$

or

$$\theta_1^I \leq \hat{\theta}_{LO-DO} \quad \text{if} \quad \hat{\theta}_{LO-DO} \leq \theta.$$  

On the other hand, \(E^D_3\) is an equilibrium if conditions (II.22) are satisfied, and in this case they can be rewritten as

$$\theta_1^I > \max\{\overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO}\} \quad \text{and} \quad \theta_2^I \leq \min\{\theta, \theta_1^I\},$$

or

$$\overline{\theta}_{LO-DO} < \theta_1^I \quad \text{and} \quad \theta < \theta_2^I \leq \theta_1^I.$$  

Notice that when \(\sigma < \kappa^I_{MO-LO} \tau\), the informed traders prefer \(LO\) to \(MO\) and, therefore, \(\overline{\theta}_{MO-DO} < \overline{\theta}_{LO-DO}\). Hence, \(E^D_3\) the equilibrium if

$$\theta_1^I > \overline{\theta}_{LO-DO} \quad \text{and} \quad \theta_2^I \leq \min\{\theta, \theta_1^I\},$$

or

$$\overline{\theta}_{LO-DO} < \theta_1^I \quad \text{and} \quad \theta < \theta_2^I \leq \theta_1^I.$$  

As a result, when \(\sigma < \kappa^I_{MO-LO} \tau\) and \(PIN \geq \psi^U_{LO-NT}\), the optimal strategy profiles at \(t = 1\) are

$$\begin{cases} 
(BLO, SLO, BLO, SLO), & \text{if} \quad \theta_1^I \leq \theta_{LO-LO}, \\
(BDO, SDO, BLO, BLO) & \text{if} \quad \theta_1^I > \theta_{DO-LO}, 
\end{cases}$$

where

$$\theta_{LO-LO}^I = \begin{cases} 
\theta_{LO-DO} & \text{if} \quad \theta < \theta_{LO-DO}, \\
\theta & \text{if} \quad \theta_{LO-DO} \leq \theta < \hat{\theta}_{LO-DO}, \quad \text{and} \\
\hat{\theta}_{LO-DO} & \text{otherwise}
\end{cases}$$

and

$$\theta_{DO-LO}^I = \begin{cases} 
\overline{\theta}_{LO-DO} & \text{if} \quad \theta_1^I \leq \min\{\theta, \theta_1^I\} \\
\overline{\theta}_{LO-DO} & \text{if} \quad \theta < \theta_2^I \leq \theta_1^I.
\end{cases}$$

**Case A.2:** \(\sigma < \kappa^I_{MO-LO} \tau\) and \(PIN \geq \psi^U_{LO-NT}\)

In this case, when we start with a market in which the equilibrium is \(E^{ND}_4\), where conditions (II.8) and (II.9) are satisfied. In this case, when we add the \(DP\) out of the 6 equilibria there are only three possible equilibria that satisfy these conditions: \(E^D_4, E^P_5, \) and \(E^D_6\).\(^2\)

\(^2\)It is straightforward that when the conditions for the equilibrium \(E^{ND}_4\) are satisfied, the equilibria \(E^D_4\) and \(E^P_5\).
In addition, we can see that $\mathcal{E}_4^D$ is an equilibrium if conditions $(II.21)$ are satisfied, and in this case they can be rewritten as

$$\theta_1^i \leq \min \{\theta, \bar{\theta}_{LO-DO}\},$$

or

$$\bar{\theta} < \theta_1^i \leq \min \{\bar{\theta}, \bar{\theta}_{LO-DO}\}.$$

Consider the following cases: I) $\bar{\theta} < \bar{\theta}_{LO-DO}$ and II) $\bar{\theta}_{LO-DO} \leq \bar{\theta}$.

**Case I:** $\bar{\theta} < \bar{\theta}_{LO-DO}$. In this case, the conditions that guarantee that $\mathcal{E}_4^D$ is an equilibrium can be rewritten as

$$\theta_1^i \leq \bar{\theta},$$

or

$$\bar{\theta} < \theta_1^i \leq \min \{\bar{\theta}, \bar{\theta}_{LO-DO}\},$$

i.e.,

$$\theta_1^i \leq \min \{\bar{\theta}, \bar{\theta}_{LO-DO}\}.$$

**Case II:** $\bar{\theta}_{LO-DO} \leq \bar{\theta}$. In this case, $\bar{\theta}_{LO-DO} < \bar{\theta}$. Thus, the conditions that guarantee that $\mathcal{E}_4^D$ is an equilibrium can be rewritten as

$$\theta_1^i \leq \bar{\theta}_{LO-DO}.$$

On the other hand, $\mathcal{E}_5^D$ is an equilibrium if conditions $(II.22)$ are satisfied, and in this case they can be rewritten as

$$\tilde{\theta}_{LO-DO} < \theta_1^i \text{ and } \bar{\theta} < \theta_2^i \leq \theta_1^i.$$ 

Finally, $\mathcal{E}_6^D$ is an equilibrium if

$$\theta_1^i > \max \{\bar{\theta}_{MO-DO}, \bar{\theta}_{LO-DO}\}, \text{ and } \theta_2^i \leq \min \{\theta, \theta_1^i\},$$

or

$$\bar{\theta}_{LO-DO} < \theta_1^i, \text{ and } \bar{\theta} < \theta_2^i \leq \min \{\bar{\theta}, \theta_1^i\},$$

Given that $\sigma < \kappa_{MO-LO}^T$, we know that the informed prefer $LO$ to $MO$. Hence, $\bar{\theta}_{LO-DO} > \bar{\theta}_{MO-DO}$, which implies $\max \{\bar{\theta}_{MO-DO}, \bar{\theta}_{LO-DO}\} = \bar{\theta}_{LO-DO}$. Thus, the previous conditions can be rewritten as

$$\theta_1^i > \bar{\theta}_{LO-DO}, \text{ and } \theta_2^i \leq \min \{\theta, \theta_1^i\},$$

or

$$\bar{\theta}_{LO-DO} < \theta_1^i, \text{ and } \bar{\theta} < \theta_2^i \leq \min \{\bar{\theta}, \theta_1^i\},$$

As a result, the optimal strategy profiles of a trader at $t = 1$ are

$$\left\{ \begin{array}{ll}
(BLO, SLO, NT, NT) & \text{if } \theta_1^i \leq \theta_{LO-NT}^{22} \\
(BDO, SDO, NT, NT) & \text{if } \theta_{DO-NT}^{22} < \theta_1^i \leq \theta_{DO-LO}^{22} \\
(BDO, SDO, BLO, BLO) & \text{if } \theta_1^i > \theta_{DO-LO}^{22},
\end{array} \right.$$ 

where

$$\theta_{LO-NT}^{22} = \left\{ \begin{array}{ll}
\min \{\bar{\theta}, \bar{\theta}_{LO-DO}\} & \text{if } \bar{\theta} < \bar{\theta}_{LO-DO} \\
\bar{\theta}_{LO-DO} & \text{otherwise,}
\end{array} \right.$$ 

are not feasible. It can be proved that in this case the equilibrium $\mathcal{E}_3^D$ is also not feasible.
\[ \theta_{DO-NT}^{22} = \begin{cases} \overline{\theta}_{LO-DO} & \text{if } \theta_2^I \leq \min \{ \theta, \theta_1^I \} \\ \tilde{\theta}_{LO-DO} & \text{if } \theta < \theta_2^I \leq \min \{ \theta, \theta_1^I \} \\ 1 & \text{if } \min \{ \theta, \theta_1^I \} < \theta_2^I, \end{cases} \] (II.25)

\[ \theta_{DO-LO}^{22} = \begin{cases} 1 & \text{if } \theta_2^I \leq \tilde{\theta}_{LO-DO} \\ \tilde{\theta}_{LO-DO} & \text{if otherwise.} \end{cases} \]

**Case A.3:** If \( \kappa_{MO-LO}^I \leq \sigma \) and \( \text{PIN} < \psi_{LO-NT}^U \)

In this case we start with a market in which the equilibrium is \( \mathcal{E}_{1^{ND}} \), where conditions (II.1) and (II.2) are satisfied. In this case, when we add the \( DP \) out of the 6 equilibria there are only two possible equilibria that satisfy these conditions: \( \mathcal{E}_1^D \) and \( \mathcal{E}_5^D \). From Lemma C.2 we can see that \( \mathcal{E}_1^D \) is an equilibrium if conditions (II.14) are satisfied. Similarly, \( \mathcal{E}_5^D \) is an equilibrium if conditions (II.22) are satisfied, and in this case they can be rewritten as

\[ \begin{align*}
\theta_1^I &> \max \{ \overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO} \} \quad \text{and} \quad \theta_2^I \leq \min \{ \theta, \theta_1^I \}, \\
or \quad 
\tilde{\theta}_{LO-DO} &< \theta_1^I \quad \text{and} \quad \theta < \theta_2^I \leq \theta_1^I.
\end{align*} \]

Given that \( \kappa_{MO-LO}^I \leq \sigma \), we have that informed traders prefer \( MO \) to \( LO \). Consequently, \( \overline{\theta}_{MO-DO} \geq \overline{\theta}_{LO-DO} \) and, therefore, we have that \( \mathcal{E}_5^D \) is an equilibrium if

\[ \begin{align*}
\theta_1^I &> \overline{\theta}_{MO-DO} \quad \text{and} \quad \theta_2^I \leq \min \{ \theta, \theta_1^I \}, \\
or \quad 
\tilde{\theta}_{LO-DO} &< \theta_1^I \quad \text{and} \quad \theta < \theta_2^I \leq \theta_1^I.
\end{align*} \]

As a result in this case the optimal strategy profiles of a trader at \( t = 1 \) are:

\[ \begin{cases} (BMO, SMO, BLO, BLO) & \text{if } \theta_1^I \leq \tilde{\theta}_{MO-DO} \\ (BDO, SDO, BLO, BLO) & \text{if } \theta_1^I > \Theta_{DO-LO}^{21} \end{cases} \]

where

\[ \Theta_{DO-LO}^{21} = \begin{cases} \overline{\theta}_{MO-DO} & \text{if } \theta_2^I \leq \min \{ \theta, \theta_1^I \} \\ \tilde{\theta}_{LO-DO} & \text{if } \theta < \theta_2^I \leq \theta_1^I. \end{cases} \] (II.26)

**Case A.4:** If \( \kappa_{MO-LO}^I \leq \sigma \) and \( \text{PIN} \geq \psi_{LO-NT}^U \)

In this case we start with a market in which the equilibrium is \( \mathcal{E}_{2^{ND}} \), where conditions (II.4) and (II.5) are satisfied. In this case, when we add the \( DP \) out of the 6 equilibria there are only three possible equilibria that satisfy these conditions: \( \mathcal{E}_2^D \), \( \mathcal{E}_5^D \), and \( \mathcal{E}_6^D \). From Lemma C.2 we can see that \( \mathcal{E}_2^D \) is an equilibrium if conditions (II.16) are satisfied. Similarly, \( \mathcal{E}_5^D \) is an equilibrium if conditions (II.22) are satisfied, and in this case they can be rewritten as

\[ \tilde{\theta}_{LO-DO} < \theta_1^I \quad \text{and} \quad \theta < \theta_2^I \leq \theta_1^I. \]

\[ ^{3}\text{It can be shown that when the conditions for the equilibrium } \mathcal{E}_1^{ND}, \text{are satisfied, the equilibrium } \mathcal{E}_3^D \text{is not feasible.} \]

\[ ^{4}\text{It can be shown that when the conditions for the equilibrium } \mathcal{E}_2^{ND}, \text{are satisfied, the equilibria } \mathcal{E}_3^D \text{and } \mathcal{E}_4^D \text{are not feasible.} \]
Finally, $E_6^D$ is an equilibrium if conditions (II.23) are satisfied, and they can be rewritten as

$$\theta_1^t > \max\{\theta_{MO-DO}, \theta_{LO-DO}\} \text{ and } \theta_2^t \leq \min\{\theta, \theta_1^t\},$$

or

$$\tilde{\theta}_{LO-DO} < \theta_1^t \text{ and } \tilde{\theta} < \theta_2^t \leq \min\{\theta, \theta_1^t\}.$$ 

Given that $\kappa_{MO-LO}^t \leq \sigma$, it follows that informed prefer $MO$ to $LO$ and, therefore, $\tilde{\theta}_{LO-DO} \leq \tilde{\theta}_{MO-DO}$, which implies $\max\{\tilde{\theta}_{MO-DO}, \tilde{\theta}_{LO-DO}\} = \tilde{\theta}_{MO-DO}$. Hence, we can rewrite them as

$$\theta_1^t > \tilde{\theta}_{MO-DO} \text{ and } \theta_2^t \leq \min\{\theta, \theta_1^t\},$$

or

$$\tilde{\theta}_{LO-DO} < \theta_1^t \text{ and } \tilde{\theta} < \theta_2^t \leq \min\{\theta, \theta_1^t\}.$$ 

As a result, the optimal strategy profiles of a trader at $t=1$ are

$$\begin{cases} 
(BMO, SMO, NT, NT) & \text{if } \theta_1^t \leq \tilde{\theta}_{MO-DO} \\
(BDO, SDO, NT, NT) & \text{if } \theta_1^t \leq \tilde{\theta}_{DO-NT} \leq \tilde{\theta}_{DO-LO} \\
(BDO, SDO, BLO, BLO) & \text{if } \theta_1^t > \theta_2^t \leq \tilde{\theta}_{DO-LO}.
\end{cases}$$

where $\theta_3^t_{DO-NT} = \begin{cases} 
\tilde{\theta}_{MO-DO} & \text{if } \theta_2^t \leq \min\{\theta, \theta_1^t\} \\
\tilde{\theta}_{LO-DO} & \tilde{\theta} < \theta_2^t \leq \min\{\theta, \theta_1^t\} \\
1 & \min\{\theta, \theta_1^t\} < \theta_2^t,
\end{cases}$

$$\theta_3^t_{DO-LO} = \begin{cases} 
1 & \text{if } \theta_2^t \leq \tilde{\theta} \\
\tilde{\theta}_{LO-DO} & \text{if } \tilde{\theta} < \theta_2^t \leq \theta_1^t.
\end{cases}$$

**Case B.** From Lemma C.2, we know that in this case there are only two possible equilibria when there is access to the $DP$: $E_2^D$ and $E_6^D$. Moreover, we have that $E_2^D$ is an equilibrium if conditions (II.16) are satisfied, while $E_6^D$ is an equilibrium if

$$\theta_1^t > \max\{\tilde{\theta}_{MO-DO}, \tilde{\theta}_{LO-DO}\} \text{ and } \theta_2^t \leq \min\{\theta, \theta_1^t\},$$

or

$$\tilde{\theta}_{LO-DO} < \theta_1^t, \text{ and } \tilde{\theta} < \theta_2^t \leq \theta_1^t.$$ 

Given that $\kappa_{MO-LO}^t < \sigma$, it follows that informed prefer $MO$ to $LO$ and, therefore, $\tilde{\theta}_{LO-DO} < \tilde{\theta}_{MO-DO}$, which implies $\max\{\tilde{\theta}_{MO-DO}, \tilde{\theta}_{LO-DO}\} = \tilde{\theta}_{MO-DO}$. Hence, we can rewrite them as

$$\theta_1^t > \tilde{\theta}_{MO-DO} \text{ and } \theta_2^t \leq \min\{\theta, \theta_1^t\},$$

or

$$\tilde{\theta}_{LO-DO} < \theta_1^t \text{ and } \tilde{\theta} < \theta_2^t \leq \theta_1^t.$$ 

As a result, the optimal strategy profiles of a trader at $t=1$ are

$$\begin{cases} 
(BMO, SMO, NT, NT) & \text{if } \theta_1^t \leq \tilde{\theta}_{MO-DO} \\
(BDO, SDO, NT, NT) & \text{if } \theta_1^t \leq \tilde{\theta}_{DO-NT} \leq \tilde{\theta}_{DO-LO} \\
\text{where } \tilde{\theta}_{DO-NT} = \begin{cases} 
\tilde{\theta}_{MO-DO} & \text{if } \theta_2^t \leq \min\{\theta, \theta_1^t\} \\
\tilde{\theta}_{LO-DO} & \tilde{\theta} < \theta_2^t \leq \min\{\theta, \theta_1^t\} \\
1 & \min\{\theta, \theta_1^t\} < \theta_2^t,
\end{cases}
\end{cases}$$

$$\text{and, therefore, } \tilde{\theta}_{DO-LO} = \begin{cases} 
\tilde{\theta}_{DO-NT} \text{ if } \tilde{\theta} \leq \theta_2^t \leq \theta_1^t.
\end{cases}$$

\[\blacksquare\]
Internet Appendix III (Proof of Propositions 3, 4, 5 and 6)

**Proof of Proposition 3.** Let us denote by $I_{t}^{ND,i}$ ($I_{t}^{D,i}$) the price informativeness in trading period $t$ corresponding to the equilibrium $\mathcal{E}_{i}^{ND}$ ($\mathcal{E}_{i}^{D}$). Note that

$$I_{1}^{ND,i} = \mathbb{E}\left[Var(V) - Var\left(V \mid (A_{2}^{ND,i}, B_{2}^{ND,i})\right)\right]$$

$$I_{2}^{ND,i} = \mathbb{E}\left[Var(V) - Var\left(V \mid (A_{2}^{ND,i}, B_{2}^{ND,i}))\right), (A_{3}^{ND,i}, B_{3}^{ND,i})\right]$$

where $(A_{2}^{ND,i}, B_{2}^{ND,i})$ and $(A_{3}^{ND,i}, B_{3}^{ND,i})$ represent the best prices at the end of period 1 and at the end of period 2, respectively, corresponding to the equilibrium $\mathcal{E}_{i}^{ND}$. Analogously, we can define $I_{1}^{D,i}$ and $I_{2}^{D,i}$.

We first compute the price informativeness at $t = 1$ for the equilibria of the single-venue market. Next, we use the following expressions:

$$Var(V) = \kappa^{2}\tau^{2},$$

$$Var\left(V \mid (A_{2}^{ND,i}, B_{2}^{ND,i})\right) = \text{pr}\left(V = V^{H} \mid (A_{2}^{ND,i}, B_{2}^{ND,i})\right)\left(V^{H} - \mathbb{E}\left(V \mid (A_{2}^{ND,i}, B_{2}^{ND,i})\right)\right)^{2} + \text{pr}\left(V = V^{L} \mid (A_{2}^{ND,i}, B_{2}^{ND,i})\right)\left(V^{L} - \mathbb{E}\left(V \mid (A_{2}^{ND,i}, B_{2}^{ND,i})\right)\right)^{2},$$

where $\text{pr}$ refers to the probability and

$$\text{pr}\left(V = V^{H} \mid (A_{2}^{ND,i}, B_{2}^{ND,i})\right) = \frac{\text{pr}\left((A_{2}^{ND,i}, B_{2}^{ND,i}) \mid V = V^{H}\right)\text{pr}(V = V^{H})}{\text{pr}\left((A_{2}^{ND,i}, B_{2}^{ND,i})\right)},$$

$$\text{pr}\left(V = V^{L} \mid (A_{2}^{ND,i}, B_{2}^{ND,i})\right) = 1 - \text{pr}\left(V = V^{H} \mid (A_{2}^{ND,i}, B_{2}^{ND,i})\right),$$

$$\mathbb{E}\left(V \mid (A_{2}^{ND,i}, B_{2}^{ND,i})\right) = \text{pr}\left(V = V^{H} \mid (A_{2}^{ND,i}, B_{2}^{ND,i})\right)V^{H} + \text{pr}\left(V = V^{L} \mid (A_{2}^{ND,i}, B_{2}^{ND,i})\right)V^{L}.$$

We compute these expressions for each equilibria of the single-venue market as follows:

<table>
<thead>
<tr>
<th>$(A_{2}^{ND,1}, B_{2}^{ND,1})$</th>
<th>$pr\left((A_{2}^{ND,1}, B_{2}^{ND,1})\right)$</th>
<th>$pr\left(V = V^{H} \mid (A_{2}^{ND,1}, B_{2}^{ND,1})\right)$</th>
<th>$pr\left(V = V^{L} \mid (A_{2}^{ND,1}, B_{2}^{ND,1})\right)$</th>
<th>$\mathbb{E}\left(V \mid (A_{2}^{ND,1}, B_{2}^{ND,1})\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(A_{1}^{1}, B_{1}^{1})$</td>
<td>$\frac{\lambda_{\tau} + 1 - \lambda}{2}$</td>
<td>$\frac{\lambda_{\tau} + 1 - \lambda}{2}$</td>
<td>$\mu + \frac{\lambda_{\tau}}{\lambda_{\tau} + 1 - \lambda} \kappa \tau$</td>
<td></td>
</tr>
<tr>
<td>$(A_{1}^{1}, B_{1}^{1} + \tau)$</td>
<td>$\frac{\lambda(1 - \tau)}{2}$</td>
<td>$\frac{\lambda}{2}$</td>
<td>$\kappa \tau$</td>
<td></td>
</tr>
<tr>
<td>$(A_{1}^{1}, B_{2}^{1})$</td>
<td>$\frac{\lambda_{\tau} + 1 - \lambda}{2}$</td>
<td>$\frac{\lambda_{\tau}}{\lambda_{\tau} + 1 - \lambda}$</td>
<td>$\mu$</td>
<td></td>
</tr>
<tr>
<td>$(A_{1}^{1} - \tau, B_{1}^{1})$</td>
<td>$\frac{\lambda(1 - \tau)}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\mu - \frac{\lambda}{\lambda_{\tau} + 1 - \lambda} \kappa \tau$</td>
<td></td>
</tr>
</tbody>
</table>
Using the previous tables, it follows that

\[ I_{1}^{ND,1} = \mathbb{E} \left( \text{Var} (V) - \text{Var} \left( V \left| \left( A_{2}^{ND,1}, B_{2}^{ND,1} \right) \right. \right) \right) = \]

\[ \begin{align*}
&= \text{pr} \left( \left( A_{2}^{1}, B_{2}^{1} \right) \right) \left( \text{Var} (V) - \text{Var} \left( V \left| \left( A_{2}^{1}, B_{2}^{1} \right) \right. \right) \right) \\
&\quad + \text{pr} \left( \left( A_{2}^{2}, B_{2}^{2} \right) \right) \left( \text{Var} (V) - \text{Var} \left( V \left| \left( A_{2}^{2}, B_{2}^{2} \right) \right. \right) \right) \\
&\quad + \text{pr} \left( \left( A_{2}^{3}, B_{2}^{3} \right) \right) \left( \text{Var} (V) - \text{Var} \left( V \left| \left( A_{2}^{3}, B_{2}^{3} \right) \right. \right) \right)
\end{align*} \]

Using the previous tables, it follows that \( I_{1}^{ND,1} = \frac{\lambda^{2} \pi^{2}}{(\lambda \pi + 1 - \lambda) \kappa^{2} \tau^{2}}. \)

In addition, note that

\[ I_{1}^{ND,2} = \mathbb{E} \left( \text{Var} (V) - \text{Var} \left( V \left| \left( A_{2}^{ND,2}, B_{2}^{ND,2} \right) \right. \right) \right) = \]

\[ \begin{align*}
&= \text{pr} \left( \left( A_{2}^{1}, B_{2}^{1} \right) \right) \left( \text{Var} (V) - \text{Var} \left( V \left| \left( A_{2}^{1}, B_{2}^{1} \right) \right. \right) \right) \\
&\quad + \text{pr} \left( \left( A_{2}^{2}, B_{2}^{2} \right) \right) \left( \text{Var} (V) - \text{Var} \left( V \left| \left( A_{2}^{2}, B_{2}^{2} \right) \right. \right) \right) \\
&\quad + \text{pr} \left( \left( A_{2}^{3}, B_{2}^{3} \right) \right) \left( \text{Var} (V) - \text{Var} \left( V \left| \left( A_{2}^{3}, B_{2}^{3} \right) \right. \right) \right)
\end{align*} \]

Using the previous tables, it follows that \( I_{1}^{ND,2} = \frac{\lambda^{2} \pi^{2}}{(\lambda \pi + 1 - \lambda) \kappa^{2} \tau^{2}}. \)
\[ \mathcal{E}_{3}^{ND} : \]

\[
\begin{array}{|c|c|c|c|}
\hline
(A_{2}^{ND,3}, B_{2}^{ND,3}) & pr \left( (A_{2}^{ND,3}, B_{2}^{ND,3}) \right) & pr \left( V = V|H \left( A_{2}^{ND,3}, B_{2}^{ND,3} \right) \right) & \mathbb{E} \left( V \left| (A_{2}^{ND,3}, B_{2}^{ND,3}) \right. \right) \\
\hline
(A_{1}^{2}, B_{1}^{1}) & \frac{1-\lambda}{2} & \frac{1}{2} & \mu \\
(A_{1}^{1}, B_{1}^{1} + \tau) & \frac{\lambda}{2} & \frac{1+\pi}{2} & \mu + \pi \kappa \tau \\
(A_{1}^{1}, B_{1}^{1}) & \frac{1-\lambda}{2} & \frac{1}{2} & \mu \\
(A_{1}^{1} - \tau, B_{1}^{1}) & \frac{\lambda}{2} & \frac{1-\pi}{2} & \mu - \pi \kappa \tau \\
\hline
\end{array}
\]

and

\[
\begin{array}{|c|c|}
\hline
(A_{2}^{ND,3}, B_{2}^{ND,3}) & \text{Var} \left( V \left| (A_{2}^{ND,3}, B_{2}^{ND,3}) \right. \right) \\
\hline
(A_{1}^{2}, B_{1}^{1}) & \kappa^{2} \tau^{2} \\
(A_{1}^{1}, B_{1}^{1} + \tau) & (1 - \pi^{2}) \kappa^{2} \tau^{2} \\
(A_{1}^{1}, B_{1}^{1}) & \kappa^{2} \tau^{2} \\
(A_{1}^{1} - \tau, B_{1}^{1}) & (1 - \pi^{2}) \kappa^{2} \tau^{2} \\
\hline
\end{array}
\]

In addition, note that

\[ I_{1}^{ND,3} = \mathbb{E} \left( \text{Var} \left( V \right) - \text{Var} \left( V \left| (A_{2}^{ND,3}, B_{2}^{ND,3}) \right. \right) \right) = \\
pr \left( (A_{2}^{2}, B_{1}^{1}) \right) \left( \text{Var} \left( V \right) - \text{Var} \left( V \left| (A_{1}^{2}, B_{1}^{1}) \right. \right) \right) + \\
pr \left( (A_{1}^{1}, B_{1}^{1} + \tau) \right) \left( \text{Var} \left( V \right) - \text{Var} \left( V \left| (A_{1}^{1}, B_{1}^{1} + \tau) \right. \right) \right) + \\
pr \left( (A_{1}^{1}, B_{1}^{1}) \right) \left( \text{Var} \left( V \right) - \text{Var} \left( V \left| (A_{1}^{1}, B_{1}^{1}) \right. \right) \right) + \\
+ pr \left( (A_{1}^{1} - \tau, B_{1}^{1}) \right) \left( \text{Var} \left( V \right) - \text{Var} \left( V \left| (A_{1}^{1} - \tau, B_{1}^{1}) \right. \right) \right). \]

Using the previous tables, it follows that \( I_{1}^{ND,3} = \lambda \pi^{2} \kappa^{2} \tau^{2} \).

\[ \mathcal{E}_{4}^{ND} : \]

\[
\begin{array}{|c|c|c|c|}
\hline
(A_{2}^{ND,4}, B_{2}^{ND,4}) & pr \left( (A_{2}^{ND,4}, B_{2}^{ND,4}) \right) & pr \left( V = V|H \left( A_{2}^{ND,4}, B_{2}^{ND,4} \right) \right) & \mathbb{E} \left( V \left| (A_{2}^{ND,4}, B_{2}^{ND,4}) \right. \right) \\
\hline
(A_{1}^{1}, B_{1}^{1}) & \lambda \left( 1 - \pi \right) & \frac{1}{2} & \mu \\
(A_{2}^{1}, B_{1}^{1}) & \frac{1-\lambda}{2} & \frac{1}{2} & \mu \\
(A_{1}^{1}, B_{1}^{1} + \tau) & \frac{\lambda}{2} & 1 & \mu + \kappa \tau \\
(A_{1}^{1}, B_{1}^{2}) & \frac{1-\lambda}{2} & \frac{1}{2} & \mu \\
(A_{1}^{1} - \tau, B_{1}^{1}) & \frac{\lambda}{2} & 0 & \mu - \kappa \tau \\
\hline
\end{array}
\]

and
In addition, note that

\[ I_1^{ND,4} = \mathbb{E} \left( \text{Var} (V) - \text{Var} \left( \frac{V}{\text{A}_2^{ND,4}, B_2^{ND,4}} \right) \right) \]

\[ = \text{pr} \left( (A_1^1, B_1^1) \right) \left( \text{Var} (V) - \text{Var} \left( \frac{V}{\text{A}_1^1, B_1^1} \right) \right) \]

\[ + \text{pr} \left( (A_2^2, B_2^1) \right) \left( \text{Var} (V) - \text{Var} \left( \frac{V}{\text{A}_2^2, B_2^1} \right) \right) + \]

\[ + \text{pr} \left( (A_1^1, B_1^1 + \tau) \right) \left( \text{Var} (V) - \text{Var} \left( \frac{V}{\text{A}_1^1 + \tau, B_1^1 + \tau} \right) \right) + \]

\[ + \text{pr} \left( (A_1^2, B_1^1) \right) \left( \text{Var} (V) - \text{Var} \left( \frac{V}{\text{A}_1^2, B_1^1} \right) \right) + \]

\[ + \text{pr} \left( (A_1^1 - \tau, B_1^1) \right) \left( \text{Var} (V) - \text{Var} \left( \frac{V}{\text{A}_1^1 - \tau, B_1^1} \right) \right). \]

Using the previous tables, it follows that \( I_1^{ND,4} = \lambda \pi \kappa^2 \tau^2. \)

In the two-venue market, if there is no order migration towards to the \( DP \) at \( t = 1 \), then the price informativeness remains unchanged, i.e., \( I_1^{D,i} = I_1^{ND,i} \), for all \( i = 1, \ldots, 4 \). For the two remaining equilibria, we have the following:

\( \mathcal{E}_5^D: \)

\[
\begin{array}{|c|c|c|}
\hline
(A_2^{D,5}, B_2^{D,5}) & \text{pr} \left( \left( A_2^{D,5}, B_2^{D,5} \right) \right) & \text{pr} \left( V = V_H | \left( A_2^{D,5}, B_2^{D,5} \right) \right) & \mathbb{E} \left( V | \left( A_2^{D,5}, B_2^{D,5} \right) \right) \\
\hline
(A_1^1, B_1^1) & \lambda \pi & \frac{1}{2} & \mu \\
\hline
(A_2^2, B_1^1) & \frac{1-\lambda}{2} & \frac{1}{2} & \mu \\
\hline
(A_1^1, B_1^1 + \tau) & \frac{\lambda (1-\pi)}{2} & \frac{1}{2} & \mu \\
\hline
(A_1^2, B_1^1) & \frac{1-\lambda}{2} & \frac{1}{2} & \mu \\
\hline
(A_1^1 - \tau, B_1^1) & \frac{\lambda (1-\pi)}{2} & \frac{1}{2} & \mu \\
\hline
\end{array}
\]

In this case, \( \text{Var} \left( \frac{V}{\text{A}_2^{D,5}, B_2^{D,5}} \right) = \text{Var} (V) \), for all \( \left( A_2^{D,5}, B_2^{D,5} \right) \). This implies that \( I_1^{D,5} = 0. \)

\( \mathcal{E}_6^D: \)

\[
\begin{array}{|c|c|c|c|}
\hline
(A_2^{D,6}, B_2^{D,6}) & \text{pr} \left( \left( A_2^{D,6}, B_2^{D,6} \right) \right) & \text{pr} \left( V = V_H | \left( A_2^{D,6}, B_2^{D,6} \right) \right) & \mathbb{E} \left( V | \left( A_2^{D,6}, B_2^{D,6} \right) \right) \\
\hline
(A_1^1, B_1^1) & \lambda \pi + \lambda (1 - \pi) = \lambda & \frac{\lambda \pi}{\lambda} = \frac{1}{2} & \mu \\
\hline
(A_2^2, B_1^1) & \frac{1-\lambda}{2} & \frac{\left(\frac{\lambda}{\lambda-\lambda}\right)}{\frac{1}{2}} = \frac{1}{2} & \mu \\
\hline
(A_1^2, B_1^1) & \frac{1-\lambda}{2} & \frac{\left(\frac{\lambda}{\lambda-\lambda}\right)}{\frac{1}{2}} = \frac{1}{2} & \mu \\
\hline
\end{array}
\]
In this case, \( Var \left(V \left( \left( A_{2}^{D,6}, B_{2}^{D,6} \right) \right) \right) = Var \left(V \right), \) for all \( \left( A_{2}^{D,6}, B_{2}^{D,6} \right) \). This implies that \( I_{1}^{D,6} = 0 \).

Finally, the comparison of price informativeness at \( t = 1 \) is summarized in the following table:

<table>
<thead>
<tr>
<th>Price Informativeness at ( t = 1 )</th>
<th>( \varepsilon_{1}^{D} )</th>
<th>( \varepsilon_{2}^{D} )</th>
<th>( \varepsilon_{3}^{D} )</th>
<th>( \varepsilon_{4}^{D} )</th>
<th>( \varepsilon_{5}^{D} )</th>
<th>( \varepsilon_{6}^{D} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_{1}^{ND} )</td>
<td>=</td>
<td>&gt;</td>
<td>=</td>
<td>&gt;</td>
<td>&gt;</td>
<td>&gt;</td>
</tr>
<tr>
<td>( \varepsilon_{2}^{ND} )</td>
<td>=</td>
<td>&gt;</td>
<td>&gt;</td>
<td>&gt;</td>
<td>&gt;</td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_{3}^{ND} )</td>
<td>=</td>
<td>&gt;</td>
<td>&gt;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_{4}^{ND} )</td>
<td>=</td>
<td>&gt;</td>
<td>&gt;</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Next, we consider the price informativeness at \( t = 2 \) without and with \( DP \) in each of the possible equilibria. The detailed calculations may be obtained on request from the authors.

<table>
<thead>
<tr>
<th>Equilibria</th>
<th>( I_{2}^{ND,3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_{1}^{ND} )</td>
<td>( 2\lambda^{2}\pi^{2} - \lambda^{2}\pi^{4} + (\lambda + 1)\lambda^{2}\pi^{2} + 2(1 - \lambda)\lambda\pi + (\lambda - 1)^{2}<em>{i,j} ) ( \kappa</em>{2}\tau_{2} )</td>
</tr>
<tr>
<td>( \varepsilon_{2}^{ND} )</td>
<td>( \frac{\lambda^{2}\pi^{2} + 2\lambda^{2}\pi^{4} - (\lambda + 1)\lambda^{2}\pi^{2} + 2(1 - \lambda)\lambda\pi + (\lambda - 1)^{2}<em>{i,j} )}{(\lambda + 1 - \lambda)^{2} - \lambda^{2} - \lambda^{2} \pi^{2}} \kappa</em>{2}\tau_{2} ) if ( X_{1}^{ND} \kappa &gt; \kappa_{2} )</td>
</tr>
<tr>
<td>( \varepsilon_{3}^{ND} )</td>
<td>( \frac{\lambda^{2}\pi^{2} + 2\lambda^{2}\pi^{4} - (\lambda + 1)\lambda^{2}\pi^{2} + 2(1 - \lambda)\lambda\pi + (\lambda - 1)^{2}<em>{i,j} )}{(\lambda + 1 - \lambda)^{2} - \lambda^{2} - \lambda^{2} \pi^{2}} \kappa</em>{2}\tau_{2} ) if ( X_{1}^{ND} \kappa \leq \kappa_{2} )</td>
</tr>
<tr>
<td>( \varepsilon_{4}^{ND} )</td>
<td>Higher than the previous one if ( Y_{2}^{ND} \kappa \leq \kappa_{1} )</td>
</tr>
</tbody>
</table>

When there is \( DP \) activity at \( t = 1 \) (i.e., \( \varepsilon_{3}^{D} \) and \( \varepsilon_{6}^{D} \)), in the case \( I_{1} \) (or, equivalently, \( \theta_{2}^{D} \leq \frac{\kappa - \lambda_{2}}{\pi_{+1} - \lambda_{2}} \)), we obtain: \( I_{2}^{D,5} = I_{2}^{D,6} = \frac{\lambda^{2}\pi^{2}}{\lambda_{+1} - \lambda} \kappa_{2}\tau_{2} \). For the remaining cases, the informational content of prices is smaller: \( I_{2}^{D,5}, I_{2}^{D,6} \leq \frac{\lambda^{2}\pi^{2}}{\lambda_{+1} - \lambda} \kappa_{2}\tau_{2} \).

We then perform a pairwise comparison of the cases (10 comparisons):

- \( \varepsilon_{1}^{ND} \) and \( \varepsilon_{5}^{D} \); \( \varepsilon_{2}^{ND} \) and \( \varepsilon_{5}^{D} \); \( \varepsilon_{2}^{ND} \) and \( \varepsilon_{6}^{D} \); \( \varepsilon_{3}^{ND} \) and \( \varepsilon_{5}^{D} \); \( \varepsilon_{4}^{ND} \) and \( \varepsilon_{5}^{D} \); \( \varepsilon_{2}^{ND} \) and \( \varepsilon_{6}^{D} \)

We find that the lower value of price informativeness corresponding to \( \varepsilon_{i}^{ND} \) is larger than the highest one corresponding to \( \varepsilon_{j}^{D} \), where \( (i, j) \in \{(1, 5), (2, 5), (2, 6), (3, 5), (4, 5), (4, 6)\} \).

Hence, \( I_{2}^{ND,1} > I_{2}^{D,1} \).

- \( \varepsilon_{1}^{ND} \) and \( \varepsilon_{1}^{D} \)

Given that in both equilibria investors behave identically, at the beginning of the second trading period, we expect the same prices. Furthermore, recall that in the second trading period, in the equilibrium \( \varepsilon_{1}^{ND} \) informed traders choose \( MO \), while uninformed traders choose \( NT \). By contrast, in the equilibrium \( \varepsilon_{1}^{D} \) informed traders choose a \( MO \) or a \( DO \), while uninformed traders choose \( NT \) or \( DO \). The fact that an informed trader at \( t = 2 \) goes to the \( DP \) leads to a reduction in price informativeness, whereas the potential change of behavior of an uninformed trader does not affect price informativeness. This implies that \( I_{2}^{ND,1} \geq I_{2}^{D,1} \).
\[ I_{2}^{ND,2} = \begin{cases} \frac{\lambda^2 \pi^2}{2} \left( 2 \lambda^2 \pi^2 (1-\lambda \pi) + (3 \lambda^2 - 4 \lambda + 5) \lambda \pi + (1-\lambda) \left( \lambda^2 - \lambda + 4 \right) \right) \kappa^2 \tau^2 & \text{if } X_2^{ND} \kappa > k_2 \\ \frac{\lambda^2 \pi^2}{2} - \lambda^3 \pi^3 + (\lambda+1) \lambda^3 \pi^2 + 2(1-\lambda) \lambda \pi + (\lambda-1)^2 \kappa^2 \tau^2 & \text{if } X_2^{ND} \kappa \leq k_2 \end{cases} \]

Moreover, it can be shown that

\[ I_{2}^{D,2} = \begin{cases} 2 \lambda^2 \pi^2 - \lambda^3 \pi^3 + (\lambda+1) \lambda^3 \pi^2 + 2(1-\lambda) \lambda \pi + (\lambda-1)^2 \kappa^2 \tau^2 & \text{in the case } U_{3}^{D} \cap I_1 \\ \frac{(1+\lambda-\pi \lambda) \lambda^2 \pi^2 \kappa^2 \tau^2}{(\lambda \pi + 1 - \lambda \pi)^2} & \text{in the case } U_{4}^{D} \cap I_3 \end{cases} \]

When we consider the parameter configuration \( X_2^{ND} > \frac{k_2}{\kappa} \) and \( U_{3}^{D} \cap I_1 \), comparing the previous expressions, it follows that \( I_{2}^{D,2} > I_{2}^{ND,2} \). Intuitively, in the equilibria \( E_2^{ND} \) and \( E_2^{D} \), an informed trader behaves the same way (submitting a \( MO \)). By contrast, for an uninformed trader we can see that in the states of the \( LOB \) in which this trader submits a \( MO \) in the single-venue market, the trader switches to a \( DO \) in the two-venue market. In the remainder states of the \( LOB \), an uninformed trader behaves equally in both equilibria. Combining these results we expect price informativeness increases when traders have access to the \( DP \), \( I_{2}^{D,2} > I_{2}^{ND,2} \).

Similarly, when we consider the parameter configuration \( X_2^{ND} > \frac{k_2}{\kappa} \) and \( U_{4}^{D} \cap I_3 \), from the previous expressions, we have \( I_{2}^{D,2} < I_{2}^{ND,2} \). Now, in the equilibria \( E_2^{ND} \) and \( E_2^{D} \), an uninformed trader behave the same way. However, while in \( E_2^{ND} \) an informed trader always submits a \( MO \), in \( E_2^{D} \) he submits either a \( MO \) or a \( DO \) depending of the state of the \( LOB \). Combining these results, we expect price informativeness increases when traders have access to the \( DP \).

**\( E_3^{ND} \) and \( E_3^{D} \)**

Given that in both equilibria investors behave identically, at the beginning of \( t = 2 \), we expect the same prices. Furthermore, recall that in the second trading period, in the equilibrium \( E_3^{ND} \) informed traders choose \( MO \), while uninformed traders choose \( MO \) or \( NT \) if \( \pi \kappa > k_1 \), and choose \( NT \) otherwise. By contrast, in the equilibrium \( E_3^{D} \) informed traders choose \( MO \) or \( DO \), while uninformed traders choose \( MO \), \( NT \) or \( DO \) if \( \pi \kappa > k_1 \), and choose \( NT \) or \( DO \) otherwise.

If \( \pi \kappa \leq k_1 \), doing a similar reasoning as the comparison of price informativeness between \( E_1^{ND} \) and \( E_1^{D} \), we conclude that \( I_{2}^{ND,3} \geq I_{2}^{D,3} \). Otherwise, i.e., if \( \pi \kappa > k_1 \), doing a similar reasoning as the comparison of the price informativeness of \( E_2^{ND} \) and \( E_2^{D} \), we obtain ambiguous results.

**\( E_4^{ND} \) and \( E_4^{D} \)**

68
Given that in both equilibria investors behave identically, at the beginning of the second trading period, we expect the same prices. Furthermore, recall that in the second trading period, in the equilibrium $E^4_{ND}$ informed traders choose $MO$, while uninformed traders choose $MO$ or $NT$. By contrast, in the equilibrium $E^4_D$ informed traders choose $MO$ or $DO$, while uninformed traders choose $NT$, $MO$ or $DO$. Doing a similar reasoning as the previous comparisons, in general, we obtain ambiguous results.

To summarize, price informativeness at $t = 2$ is as follows:

<table>
<thead>
<tr>
<th>Price Informativeness $t = 2$</th>
<th>$\mathcal{E}^D_1$</th>
<th>$\mathcal{E}^D_2$</th>
<th>$\mathcal{E}^D_3$</th>
<th>$\mathcal{E}^D_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{E}^{ND}_1$</td>
<td>$\geq$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{E}^{ND}_2$</td>
<td>$\gg$</td>
<td>$&gt;$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{E}^{ND}_3$</td>
<td>$\gg$</td>
<td>$&gt;$</td>
<td>$&gt;$</td>
<td></td>
</tr>
<tr>
<td>$\mathcal{E}^{ND}_4$</td>
<td></td>
<td>$&gt;$</td>
<td>$&gt;$</td>
<td>$&gt;$</td>
</tr>
</tbody>
</table>

Proof of Proposition 4. We first compute the ex-ante expected inside spread in the exchange for the equilibria in the single-venue market. In the following table, for each couple of best prices of the LOB at the end of $t = 1$, it is displayed the value of inside spread, $S_1$, and its corresponding probability for each equilibrium.

<table>
<thead>
<tr>
<th>$(A^1_i, B^1_i)$</th>
<th>$S_1$</th>
<th>$\text{prob}^{E^ND}_1$</th>
<th>$\text{prob}^{E^ND}_2$</th>
<th>$\text{prob}^{E^ND}_3$</th>
<th>$\text{prob}^{E^ND}_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(A^1_1, B^1_1)$</td>
<td>$2k_1\tau$</td>
<td>$0$</td>
<td>$\lambda(1 - \pi)$</td>
<td>$0$</td>
<td>$\lambda(1 - \pi)$</td>
</tr>
<tr>
<td>$(A^2_1, B^1_1)$</td>
<td>$(k_2 + k_1)\tau$</td>
<td>$\lambda$</td>
<td>$\frac{k_2}{2} + \frac{1 - \lambda}{2}$</td>
<td>$\lambda$</td>
<td>$\frac{k_2}{2} + \frac{1 - \lambda}{2}$</td>
</tr>
<tr>
<td>$(A^1_1, B^1_1 + \tau)$</td>
<td>$(2k_1 - 1)\tau$</td>
<td>$\frac{\lambda(1 - \pi)}{2}$</td>
<td>$0$</td>
<td>$\frac{\lambda}{2}$</td>
<td>$\frac{\lambda}{2}$</td>
</tr>
<tr>
<td>$(A^1_2, B^1_1)$</td>
<td>$(k_2 + k_1)\tau$</td>
<td>$\lambda$</td>
<td>$\frac{k_2}{2} + \frac{1 - \lambda}{2}$</td>
<td>$\lambda$</td>
<td>$\frac{k_2}{2} + \frac{1 - \lambda}{2}$</td>
</tr>
<tr>
<td>$(A^1_1 - \tau, B^1_1)$</td>
<td>$(2k_1 - 1)\tau$</td>
<td>$\frac{\lambda(1 - \pi)}{2}$</td>
<td>$0$</td>
<td>$\frac{\lambda}{2}$</td>
<td>$\frac{\lambda}{2}$</td>
</tr>
</tbody>
</table>

Using the values included in the previous table, it follows that the expected spreads at $t = 1$ in each equilibrium equal

$$
\mathbb{E}_0\left(S^{E^ND}_1\right) = (2k_1 + (1 - \lambda)(1 - \pi)(k_2 - k_1)) - \lambda(1 - \pi) \tau,
$$

$$
\mathbb{E}_0\left(S^{E^ND}_1\right) = (2k_1 + (1 - \lambda)(1 - \pi)(k_2 - k_1)) \tau,
$$

$$
\mathbb{E}_0\left(S^{E^ND}_1\right) = (2k_1 + (1 - \lambda)(k_2 - k_1) - \lambda) \tau, \text{ and}
$$

$$
\mathbb{E}_0\left(S^{E^ND}_1\right) = (2k_1 + (1 - \lambda)(k_2 - k_1) - \lambda \pi) \tau.
$$

With access to the DP, if there is no order migration towards to the $DP$ at $t = 1$, then the ex-ante inside spread remains unchanged, i.e., $\mathbb{E}_0\left(S^{E^ND}_1\right) = \mathbb{E}_0\left(S^{E^ND}_1\right), \text{ for all } i = 1, ..., 4$. For the two remaining equilibria, the following table shows the value of inside spread and its corresponding probability for each couple of best prices:
From the previous table, it follows that

\[ E_0 \left( S_{E^D} \right) = (2k_1 + (1 - \lambda)(k_2 - k_1) - \lambda(1 - \pi)) \tau \] and

\[ E_0 \left( S_{E^N} \right) = (2k_1 + (1 - \lambda)(k_2 - k_1)) \tau. \]

The change in inside spread due to the introduction of the DP is

\[ E_0 \left( S_{E^D} \right) - E_0 \left( S_{E^N} \right) \]

<table>
<thead>
<tr>
<th>( E_0 \left( S_{E^D} \right) - E_0 \left( S_{E^N} \right) )</th>
<th>( \xi_1^D )</th>
<th>( \xi_2^D )</th>
<th>( \xi_3^D )</th>
<th>( \xi_4^D )</th>
<th>( \xi_5^D )</th>
<th>( \xi_6^D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_1^{ND} )</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \xi_2^{ND} )</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \xi_3^{ND} )</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \xi_4^{ND} )</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Therefore, at \( t = 1 \) we have the following relationship between spreads in the two-venue market and the single-venue market.

We consider next the ex-ante expected inside spread at \( t = 2 \) without and with DP in each of the possible equilibria. The calculations supporting the following tables for ex-ante expected inside spread without the DP and with the DP can be obtained on request from the authors. In what follows, \( I_1 - I_6 \) and \( U_i^{E^D} \), \( i = 1, \ldots, 6 \), correspond to the cases of the equilibria in the two-venue market given in Internet Appendix II.

The ex-ante expected inside spread without the DP in each equilibria equals
<table>
<thead>
<tr>
<th>Equilibria</th>
<th>( \mathbb{E}<em>0 \left( S</em>{2}^{E_{i}} \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{E}_{1}^{ND} )</td>
<td>( \Psi_{ND,1} )</td>
</tr>
<tr>
<td>( \mathcal{E}<em>{2}^{ND}, X^{2,ND} &gt; k</em>{2}/\kappa )</td>
<td>( \Psi_{ND,2} )</td>
</tr>
<tr>
<td>( \mathcal{E}<em>{2}^{ND}, X^{2,ND} \leq k</em>{2}/\kappa )</td>
<td>( \Psi_{ND,2} - \lambda (1-\pi)(1-\lambda+\lambda\pi)(k_{3}-k_{2}) \tau )</td>
</tr>
<tr>
<td>( \mathcal{E}<em>{3}^{ND}, Y^{3,ND} &gt; k</em>{1}/\kappa )</td>
<td>( \Psi_{ND,3} )</td>
</tr>
<tr>
<td>( \mathcal{E}<em>{3}^{ND}, Y^{3,ND} \leq k</em>{1}/\kappa )</td>
<td>( \Psi_{ND,3} - \lambda (1-\pi)(k_{3}-k_{1}) \tau )</td>
</tr>
<tr>
<td>( \mathcal{E}_{4}^{ND} )</td>
<td>( \Psi_{ND,4} )</td>
</tr>
</tbody>
</table>

where

\[
\Psi_{ND,1} = 4k_{1} + ((1-\lambda)^{2}+2\lambda\pi(1-\lambda+\lambda\pi))(k_{3}-k_{2}) + (3(1-\lambda+\lambda\pi) - \pi^{2}\lambda^{2})(k_{2}-k_{1}) - \lambda (1-\pi)(1-\pi+1) \tau,
\]

\[
\Psi_{ND,2} = \frac{4k_{1} + ((1-\lambda+\lambda\pi+\lambda^{2}\pi^{2})(k_{3}-k_{2}) + (3-\lambda^{2} - 2\pi(1-\pi)(1-\pi))(k_{2}-k_{1})}{2} \tau,
\]

\[
\Psi_{ND,3} = 4k_{1} + (1-\lambda)(1-\lambda+\lambda\pi)(k_{3}-k_{2}) + (3-2\lambda+\lambda^{2} - \lambda(1-\pi)(1+1))(k_{2}-k_{1}) - \lambda (1+\lambda+\lambda\pi+\lambda\pi^{2}) \tau,
\]

and

\[
\Psi_{ND,4} = \frac{4k_{1} + (1-\lambda)(1-\lambda+\lambda\pi)(k_{3}-k_{2}) + (3-2\lambda+\lambda^{2} - \lambda^{2}(1-\pi)(2-\pi))(k_{2}-k_{1}) - \lambda (1+\lambda)(1+\lambda) \tau}{2}.
\]

The ex-ante expected inside spread with the DP in each equilibria equals

<table>
<thead>
<tr>
<th>Cases</th>
<th>( \mathbb{E}<em>0 \left( S</em>{2}^{E_{i}} \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>{ ( U_{1}^{E_{1}} ), ( U_{2}^{E_{2}} ), ( U_{3}^{E_{3}} ) } &amp; ( I_{1} )</td>
<td>( \Psi_{ND,1} - \lambda (1-\lambda+\lambda\pi)(k_{3}-k_{2}) \tau )</td>
</tr>
<tr>
<td>{ ( U_{1}^{E_{1}} ), ( U_{2}^{E_{2}} ), ( U_{3}^{E_{3}} ) } &amp; ( I_{2} )</td>
<td>( \Psi_{ND,2} - \lambda (1-\lambda)(1-\lambda+\lambda\pi)(k_{3}-k_{2}) \tau + \lambda (1+\lambda)(k_{3}-k_{2}) \tau )</td>
</tr>
<tr>
<td>{ ( U_{1}^{E_{1}} ), ( U_{2}^{E_{2}} ), ( U_{3}^{E_{3}} ) } &amp; ( I_{3} )</td>
<td>( \Psi_{ND,2} - \lambda (1-\lambda+\lambda\pi)(k_{3}-k_{2}) \tau + \lambda (1-\lambda)(k_{3}-k_{1}) + 2\lambda (3-2\lambda+\lambda^{2}) \tau )</td>
</tr>
<tr>
<td>( U_{4}^{E_{4}} ) &amp; ( I_{1} )</td>
<td>( \Psi_{ND,2} )</td>
</tr>
<tr>
<td>( U_{4}^{E_{4}} ) &amp; ( I_{2} )</td>
<td>( \Psi_{ND,2} - \lambda (1-\lambda+\lambda\pi)(k_{3}-k_{2}) \tau )</td>
</tr>
<tr>
<td>( U_{4}^{E_{4}} ) &amp; ( I_{3} )</td>
<td>( \Psi_{ND,2} - \lambda (1-\lambda)(k_{3}-k_{1}) + 2\lambda (3-2\lambda+\lambda^{2}) \tau )</td>
</tr>
<tr>
<td>Cases</td>
<td>( E_0 \left( S_2^{E_T'} \right) )</td>
</tr>
<tr>
<td>-------</td>
<td>------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>( { U_{1,1}^{E_T}, U_{1,2}^{E_T}, U_{1,3}^{E_T} } ) &amp; ( I_1 )</td>
<td>( \Psi_{N,D,3} - \frac{\lambda^2(1-\pi)(k_2-k_1)}{2} \tau )</td>
</tr>
<tr>
<td>( { U_{1,1}^{E_T}, U_{1,2}^{E_T}, U_{1,3}^{E_T} } ) &amp; ( { I_2, I_3, I_4 } )</td>
<td>( \Psi_{N,D,3} - \frac{\lambda^2(1-\pi)(k_2-k_1)}{2} \tau - \frac{\lambda\pi(1-\lambda)(k_3-k_2)}{2} \tau )</td>
</tr>
<tr>
<td>( { U_{1,1}^{E_T}, U_{1,2}^{E_T}, U_{1,3}^{E_T} } ) &amp; ( I_5 )</td>
<td>( \Psi_{N,D,3} - \frac{\lambda^2(1-\pi)(k_2-k_1)}{2} \tau - \frac{\lambda\pi((1-\lambda)(k_3-k_1)+\lambda(1+\pi)(k_2-k_1))}{2} \tau )</td>
</tr>
<tr>
<td>( { U_{1,1}^{E_T}, U_{1,2}^{E_T}, U_{1,3}^{E_T} } ) &amp; ( I_6 )</td>
<td>( \Psi_{N,D,3} - \frac{\lambda^2(1-\pi)(k_2-k_1)}{2} \tau - \frac{\lambda\pi((1-\lambda)(k_3-k_1)+\lambda(1+\pi)(k_2-k_1)+\lambda(1-\pi))}{2} \tau )</td>
</tr>
<tr>
<td>( U_{4,1}^{E_T} ) &amp; ( I_1 )</td>
<td>( \Psi_{N,D,3} )</td>
</tr>
<tr>
<td>( U_{4,1}^{E_T} ) &amp; ( I_2 )</td>
<td>( \Psi_{N,D,4} - \frac{\lambda\pi(1-\lambda)(k_3-k_2)}{2} \tau )</td>
</tr>
<tr>
<td>( U_{4,1}^{E_T} ) &amp; ( I_3 )</td>
<td>( \Psi_{N,D,4} - \frac{\lambda\pi(1-\lambda)(k_3-k_2)}{2} \tau )</td>
</tr>
<tr>
<td>( U_{4,1}^{E_T} ) &amp; ( I_4 )</td>
<td>( \Psi_{N,D,4} - \frac{\lambda\pi((1-\lambda)(k_3-k_1)+2\lambda(k_2-k_1)(1-\pi))}{2} \tau )</td>
</tr>
<tr>
<td>( U_{4,1}^{E_T} ) &amp; ( I_5 )</td>
<td>( \Psi_{N,D,4} - \frac{\lambda\pi((1-\lambda)(k_3-k_1)+2\lambda(k_2-k_1)(1-\pi))}{2} \tau )</td>
</tr>
</tbody>
</table>

\[
\Psi_{D,5} = \frac{4k_1+(1-\lambda)(1-\lambda+\lambda\pi)(k_3-k_2)+(3-3\lambda+\lambda\pi(2-\lambda+\lambda\pi))(k_2-k_1)-\lambda(1-\pi)(1+\lambda-\lambda\pi)}{2} \tau
\]

\[
\Psi_{D,6} = \frac{4k_1+(1-\lambda)(1-\lambda+\lambda\pi)(k_3-k_2)+(3-3\lambda-\lambda\pi(1+\lambda)(1-\pi))(k_2-k_1)}{2} \tau
\]
We then perform a pairwise comparison of the cases (10 comparisons):

- **$E_{1}^{ND}$ and $E_{1}^{D}$; $E_{4}^{ND}$ and $E_{4}^{D}$**

  Using the information of the relevant tables, we notice that at $t = 2$ the ex-ante expected inside spread corresponding to $E_{i}^{ND}$ is equal to the highest one corresponding to $E_{j}^{D}$, where $(i, j) \in \{(1, 1), (4, 4)\}$. Therefore, we conclude that $\mathbb{E}_{0}\left(S_{2}^{E_{i}^{ND}}\right) \geq \mathbb{E}_{0}\left(S_{2}^{E_{j}^{D}}\right)$.

- **$E_{2}^{ND}$ and $E_{2}^{D}$**

  We distinguish two cases: 1) $X_{2,ND}^{2} = \frac{\lambda \pi}{1 - \lambda + \lambda \pi} > \frac{k_{2}}{\kappa}$, and 2) $X_{2,ND}^{2} = \frac{\lambda \pi}{1 - \lambda + \lambda \pi} \leq \frac{k_{2}}{\kappa}$. In this case, we find that the ex-ante expected inside spread corresponding to $E_{2}^{ND}$ is equal to the highest one corresponding to $E_{2}^{D}$. Hence, $\mathbb{E}_{0}\left(S_{2}^{E_{2}^{ND}}\right) \geq \mathbb{E}_{0}\left(S_{2}^{E_{2}^{D}}\right)$.

- **$E_{3}^{ND}$ and $E_{3}^{D}$**

  We can distinguish 2 cases:
  
  Case 1: $Y_{3,ND}^{3} = \pi > \frac{k_{1}}{\kappa}$. We find that the ex-ante expected inside spread corresponding to $E_{3}^{ND}$ is equal to the highest one corresponding to $E_{3}^{D}$. Hence, $\mathbb{E}_{0}\left(S_{2}^{E_{3}^{ND}}\right) \geq \mathbb{E}_{0}\left(S_{2}^{E_{3}^{D}}\right)$.
  
  Case 2: $Y_{3,ND}^{3} = \pi \leq \frac{k_{1}}{\kappa}$. It is important to point out that in this case, the cases when there is access to the $DP$ which are compatible with the previous inequality ($X_{2,ND}^{2} \leq \frac{k_{2}}{\kappa}$) are $U_{1}^{E_{2}^{D}}, U_{2}^{E_{2}^{D}}$ and $U_{3}^{E_{2}^{D}}$. Then, the ex-ante expected inside spread corresponding to $E_{3}^{ND}$ is equal to the highest one corresponding to $E_{2}^{D}$. Hence, $\mathbb{E}_{0}\left(S_{2}^{E_{3}^{ND}}\right) \geq \mathbb{E}_{0}\left(S_{2}^{E_{2}^{D}}\right)$.

- **$E_{1}^{ND}$ and $E_{5}^{D}$; $E_{2}^{ND}$ and $E_{3}^{D}$**

  Using the information of the relevant tables, we find that the ex-ante expected inside spread corresponding to $E_{i}^{ND}$ is larger than the highest one corresponding to $E_{j}^{D}$, where $(i, j) \in \{(1, 5), (2, 5), (2, 6)\}$. Hence, $\mathbb{E}_{0}\left(S_{2}^{E_{i}^{ND}}\right) > \mathbb{E}_{0}\left(S_{2}^{E_{j}^{D}}\right)$.

- **$E_{3}^{ND}$ and $E_{5}^{D}$**
Using the information of the relevant tables, we find that the lowest ex-ante expected inside spread corresponding to $E_3^{ND}$ is smaller than the highest one corresponding to $E_5^D$, while the highest ex-ante expected inside spread corresponding to $E_3^{ND}$ is higher than the lowest one corresponding to $E_5^D$. Consequently, this comparison is in general ambiguous. Given that this result differs from the previous ones, let us to prove a numerical example in which we expect a higher inside spread at $t = 2$ with the access of $DP$. Suppose that $\lambda = 0.2$, $\pi = 1/2$, $\delta = 1$, $\kappa = 6$, $k_3 = 5$, $k_2 = 4$, $k_1 = 3$, $\theta_1^I = 0.75$, and $\theta_2^I = 0.25$. The equilibrium without the $DP$ is $E_3^{ND}$, while with the $DP$ is $E_5^D$. At $t = 1$ the introduction of a $DP$ leads to a migration of $LO$ set by informed traders to the $DO$, which implies an increase in the spreads at $t = 1$. At $t = 2$, given that $\theta_2^I \leq \frac{6-\pi}{6} = 0.36364$, in $E_5^D$ informed traders choose market orders, while uninformed traders choose either not to trade or $DO$. On the other hand, in $E_3^{ND}$ informed traders choose $MO$, while uninformed traders choose $NT$ since $\pi = \frac{k_1}{\kappa}$. As the behavior of rational traders at $t = 2$ modifies in the same way the spreads, we expect that the spreads at $t = 2$ to continue to be higher when the $DP$ is available.

**$E_4^{ND}$ and $E_5^D$**

Using the relevant tables and taking into account that the migration from $E_4^{ND}$ to $E_5^D$ only has sense in the case $I_6$. Direct computations yield $E_0 \left( S_2^{E_2^{ND}} \right) > E_0 \left( S_2^{E_5^D} \right)$.

**$E_4^{ND}$ and $E_6^D$**

Using the relevant tables, we first find that the ex-ante expected inside spread corresponding to $E_4^{ND}$ is higher than the lowest one corresponding to $E_6^D$. In relation to the difference between the ex-ante expected inside spread corresponding to $E_4^{ND}$ and the highest one corresponding to $E_6^D$, we find that the difference is equal to the following expression: $\frac{1}{2} \lambda \pi \left( -\lambda - 1 + (k_2 - k_1) (-\lambda \pi + 2\lambda - 1) \right) \tau$. Hence, we can derive two cases: 1) $(-\lambda \pi + 2\lambda - 1) \leq 0$ or $(-\lambda \pi + 2\lambda - 1) > 0$ and $k_2 < \frac{1+\lambda}{\lambda \pi + 2\lambda - 1} + k_1$, and 2) $(-\lambda \pi + 2\lambda - 1) > 0$ and $k_2 \geq \frac{1+\lambda}{\lambda \pi + 2\lambda - 1} + k_1$.

- If $(-\lambda \pi + 2\lambda - 1) \leq 0$ or $(-\lambda \pi + 2\lambda - 1) > 0$ and $k_2 < \frac{1+\lambda}{\lambda \pi + 2\lambda - 1} + k_1$ (i.e. is relatively small) then the ex-ante expected inside spread corresponding to $E_4^{ND}$ is lower than the highest one corresponding to $E_6^D$. Hence, there are parameter values such that $E_0 \left( S_{2}^{E_4^{ND}} \right) < E_0 \left( S_{2}^{E_6^D} \right)$.

- If $(-\lambda \pi + 2\lambda - 1) > 0$ and $k_2 \geq \frac{1+\lambda}{\lambda \pi + 2\lambda - 1} + k_1$, is sufficiently high, then the ex-ante expected inside spread corresponding to $E_4^{ND}$ is higher than the one corresponding to $E_6^D$. Hence, there are parameter values such that $E_0 \left( S_{2}^{E_4^{ND}} \right) > E_0 \left( S_{2}^{E_6^D} \right)$.

Finally, in the following table we sum up the results on the comparison of the inside spread at $t = 2$ when going from the two-venue model to an equilibrium with the single-venue model.
Proof of Proposition 5. We first compute ex-ante expected volume at \( t = 1 \) in the exchange for the equilibria for the single-venue market. The following table displays, for each equilibrium, the value of trading volume in the exchange and its corresponding probability, for each couple of best prices of the LOB at the end of \( t = 1 \):

<table>
<thead>
<tr>
<th>( E_D^1 )</th>
<th>( \mathcal{E}_{1}^{\mathcal{N}D} )</th>
<th>( \mathcal{E}_{2}^{\mathcal{N}D} )</th>
<th>( \mathcal{E}_{3}^{\mathcal{N}D} )</th>
<th>( \mathcal{E}_{4}^{\mathcal{N}D} )</th>
<th>( \mathcal{E}_{5}^{\mathcal{N}D} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_D^{1} )</td>
<td>( \geq )</td>
<td>( &gt; )</td>
<td>( &gt; )</td>
<td>( &gt; )</td>
<td>( &gt; )</td>
</tr>
<tr>
<td>( E_D^{2} )</td>
<td>( \geq )</td>
<td>( &gt; )</td>
<td>( &gt; )</td>
<td>( &gt; )</td>
<td>( &gt; )</td>
</tr>
<tr>
<td>( E_D^{3} )</td>
<td>( &gt; )</td>
<td>( &gt; )</td>
<td>( &gt; )</td>
<td>( &gt; )</td>
<td>( &gt; )</td>
</tr>
<tr>
<td>( E_D^{4} )</td>
<td>( &gt; )</td>
<td>( &gt; )</td>
<td>( &gt; )</td>
<td>( &gt; )</td>
<td>( &gt; )</td>
</tr>
</tbody>
</table>

Hence, it follows that

\[
\begin{align*}
\mathbb{E}_0 \left( V_{E_X,1}^{\mathcal{N}D} \right) &= \mathbb{E}_0 \left( V_{E_X,1}^{\mathcal{E}_1^{\mathcal{N}D}} \right) = 1 - \lambda + \lambda \pi \\
\mathbb{E}_0 \left( V_{E_X,1}^{\mathcal{N}D} \right) &= \mathbb{E}_0 \left( V_{E_X,1}^{\mathcal{E}_2^{\mathcal{N}D}} \right) = 1 - \lambda.
\end{align*}
\]

When traders have access to the DP, we have to take into account that trade can take place in one of the two venues. Thus, we calculate the ex-ante expected trading volume in the exchange, \( \mathbb{E}_0 \left( V_{E_X,1}^{\mathcal{N}D} \right) \), the trading volume in the DP, \( \mathbb{E}_0 \left( V_{D_P,1}^{\mathcal{N}D} \right) \), and its sum, which is the total ex-ante expected trading volume at \( t = 1 \), \( \mathbb{E}_0 \left( V_{T,1}^{\mathcal{N}D} \right) \), for all the equilibria \( \mathcal{E}_i^{\mathcal{N}D} \), \( i = 1, ..., 6 \).

In relation to the expected trading volume in the exchange, notice that \( \mathbb{E}_0 \left( V_{E_X,1}^{\mathcal{N}D} \right) = \mathbb{E}_0 \left( V_{E_X,1}^{\mathcal{E}_i^{\mathcal{N}D}} \right) \), for \( i = 1, ..., 4 \). For the two other equilibria (i.e., \( \mathcal{E}_5^{\mathcal{N}D} \) and \( \mathcal{E}_6^{\mathcal{N}D} \)), the value of trading volume in the exchange and its corresponding probability are the following:

<table>
<thead>
<tr>
<th>( V_{E_X,1} )</th>
<th>prob( \mathcal{E}_5^{\mathcal{N}D} )</th>
<th>prob( \mathcal{E}_6^{\mathcal{N}D} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (A_1^1, B_1^1) )</td>
<td>0</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>( (A_1^2, B_1^1) )</td>
<td>1</td>
<td>( \frac{1-\lambda}{2} )</td>
</tr>
<tr>
<td>( (A_1^1, B_1^1 + \tau) )</td>
<td>0</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>( (A_1^1, B_1^1 - \tau) )</td>
<td>0</td>
<td>( \lambda )</td>
</tr>
</tbody>
</table>
Again, taking expectations, we obtain that

$$E_0\left(V_{E_X,1}\right) = 1 - \lambda.$$ 

The ex-ante expected trading volume in the DP yields $E_0\left(V_{E_D,i}\right) = 0$, for $i = 1, ..., 4$, and $E_0\left(V_{E_D,i}\right) = \lambda \pi \theta_1^i$, for $i = 5, 6$. The following table summarizes the ex-ante expected trading volume in the exchange, in the DP and the total trading volume:

<table>
<thead>
<tr>
<th></th>
<th>$\mathcal{E}_1^D$</th>
<th>$\mathcal{E}_2^D$</th>
<th>$\mathcal{E}_3^D$</th>
<th>$\mathcal{E}_4^D$</th>
<th>$\mathcal{E}_5^D$</th>
<th>$\mathcal{E}_6^D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_0\left(V_{E_X,1}\right)$</td>
<td>$1 - \lambda + \lambda \pi$</td>
<td>$1 - \lambda + \lambda \pi$</td>
<td>$1 - \lambda$</td>
<td>$1 - \lambda$</td>
<td>$1 - \lambda$</td>
<td>$1 - \lambda$</td>
</tr>
<tr>
<td>$E_0\left(V_{E_D,1}\right)$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$\lambda \pi \theta_1^1$</td>
<td>$\lambda \pi \theta_1^1$</td>
</tr>
<tr>
<td>$E_0\left(V_{E_D,2}\right)$</td>
<td>$1 - \lambda + \lambda \pi$</td>
<td>$1 - \lambda + \lambda \pi$</td>
<td>$1 - \lambda$</td>
<td>$1 - \lambda$</td>
<td>$1 - \lambda + \lambda \pi \theta_1^1$</td>
<td>$1 - \lambda + \lambda \pi \theta_1^1$</td>
</tr>
</tbody>
</table>

Direct computations yield the comparison of ex-ante expected trading volume in the exchange and in the DP at $t = 1$:

<table>
<thead>
<tr>
<th></th>
<th>$\mathcal{E}_1^D$</th>
<th>$\mathcal{E}_2^D$</th>
<th>$\mathcal{E}_3^D$</th>
<th>$\mathcal{E}_4^D$</th>
<th>$\mathcal{E}_5^D$</th>
<th>$\mathcal{E}_6^D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_0\left(V_{E_X,1}\right)$</td>
<td>$\mathcal{E}_1^{ND}$</td>
<td>$\mathcal{E}_2^{ND}$</td>
<td>$\mathcal{E}_3^{ND}$</td>
<td>$\mathcal{E}_4^{ND}$</td>
<td>$\mathcal{E}_5^{ND}$</td>
<td>$\mathcal{E}_6^{ND}$</td>
</tr>
<tr>
<td>$E_0\left(V_{E_D,1}\right)$</td>
<td>$\mathcal{E}_1^{ND}$</td>
<td>$\mathcal{E}_2^{ND}$</td>
<td>$\mathcal{E}_3^{ND}$</td>
<td>$\mathcal{E}_4^{ND}$</td>
<td>$\mathcal{E}_5^{ND}$</td>
<td>$\mathcal{E}_6^{ND}$</td>
</tr>
<tr>
<td>$E_0\left(V_{E_D,2}\right)$</td>
<td>$\mathcal{E}_1^{ND}$</td>
<td>$\mathcal{E}_2^{ND}$</td>
<td>$\mathcal{E}_3^{ND}$</td>
<td>$\mathcal{E}_4^{ND}$</td>
<td>$\mathcal{E}_5^{ND}$</td>
<td>$\mathcal{E}_6^{ND}$</td>
</tr>
<tr>
<td>$E_0\left(V_{E_D,2}\right)$</td>
<td>$\mathcal{E}_1^{ND}$</td>
<td>$\mathcal{E}_2^{ND}$</td>
<td>$\mathcal{E}_3^{ND}$</td>
<td>$\mathcal{E}_4^{ND}$</td>
<td>$\mathcal{E}_5^{ND}$</td>
<td>$\mathcal{E}_6^{ND}$</td>
</tr>
</tbody>
</table>

Moreover, in relation to the ex-ante total expected volume at $t = 1$, we have that

<table>
<thead>
<tr>
<th></th>
<th>$\mathcal{E}_1^D$</th>
<th>$\mathcal{E}_2^D$</th>
<th>$\mathcal{E}_3^D$</th>
<th>$\mathcal{E}_4^D$</th>
<th>$\mathcal{E}_5^D$</th>
<th>$\mathcal{E}_6^D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_0\left(V_{E_X,1}\right)$</td>
<td>$\mathcal{E}_1^{ND}$</td>
<td>$\mathcal{E}_2^{ND}$</td>
<td>$\mathcal{E}_3^{ND}$</td>
<td>$\mathcal{E}_4^{ND}$</td>
<td>$\mathcal{E}_5^{ND}$</td>
<td>$\mathcal{E}_6^{ND}$</td>
</tr>
<tr>
<td>$E_0\left(V_{E_D,1}\right)$</td>
<td>$\mathcal{E}_1^{ND}$</td>
<td>$\mathcal{E}_2^{ND}$</td>
<td>$\mathcal{E}_3^{ND}$</td>
<td>$\mathcal{E}_4^{ND}$</td>
<td>$\mathcal{E}_5^{ND}$</td>
<td>$\mathcal{E}_6^{ND}$</td>
</tr>
<tr>
<td>$E_0\left(V_{E_D,2}\right)$</td>
<td>$\mathcal{E}_1^{ND}$</td>
<td>$\mathcal{E}_2^{ND}$</td>
<td>$\mathcal{E}_3^{ND}$</td>
<td>$\mathcal{E}_4^{ND}$</td>
<td>$\mathcal{E}_5^{ND}$</td>
<td>$\mathcal{E}_6^{ND}$</td>
</tr>
</tbody>
</table>

We then consider the ex-ante expected trading volume at $t = 2$ in the exchange, in the DP and total volume of trading in each of the possible equilibria when there is a DP. The calculations supporting the following tables for expected volume without the DP and with the DP can be obtained on request from the authors. Again, in what follows, $I_1 - I_6$ and $U_{j,i}^D$, $i = 1, ..., 6$, correspond to the cases referred to in the analysis performed to derive the equilibria when there is access to
the $DP$ given in Internet Appendix II.

Thus, the ex-ante expected trading volume without the $DP$ in each equilibrium equals

<table>
<thead>
<tr>
<th>Equilibria</th>
<th>$E_0 \left( V^{E\text{X}}_{EX,2} \right)$</th>
<th>$E_0 \left( V^{E\text{X}}_{D\text{P},2} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1^{ND}$</td>
<td>$(1 - \lambda + \lambda \pi)$</td>
<td>$\lambda (1 - \pi) \theta_1^U$</td>
</tr>
<tr>
<td>$E_2^{ND}$, $X^{2,ND} &gt; k_2/\kappa$</td>
<td>$(1 - \lambda + \lambda \pi) \left( 1 + \frac{\lambda(1 - \pi)}{2} \right)$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda + \frac{1 - \lambda \pi}{2} \right) \theta_1^I$</td>
</tr>
<tr>
<td>$E_3^{ND}$, $X^{2,ND} \leq k_2/\kappa$</td>
<td>$(1 - \lambda + \lambda \pi)$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda - \frac{1 - \lambda \pi}{2} \right) \theta_1^I$</td>
</tr>
<tr>
<td>$E_4^{ND}$, $Y^{3,ND} &gt; k_1/\kappa$</td>
<td>$(1 - \lambda + \lambda \pi + \frac{\lambda^2 \pi (1 - \pi)}{2})$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda - \frac{1 - \lambda \pi}{2} \right) \theta_1^I$</td>
</tr>
<tr>
<td>$E_5^{ND}$, $Y^{3,ND} \leq k_1/\kappa$</td>
<td>$(1 - \lambda + \lambda \pi + \frac{\lambda^2 \pi (1 - \pi)}{2})$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda - \frac{1 - \lambda \pi}{2} \right) \theta_1^I$</td>
</tr>
</tbody>
</table>

Similarly, the ex-ante expected trading volume with the $DP$

<table>
<thead>
<tr>
<th>Cases</th>
<th>$E_0 \left( V^{E\text{X}}_{EX,2} \right)$</th>
<th>$E_0 \left( V^{E\text{X}}_{D\text{P},2} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${U_1^{F1}, U_3^{F1}} &amp; I_1$</td>
<td>$(1 - \lambda + \lambda \pi)$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda - \frac{1 - \lambda \pi}{2} \right) \theta_1^I$</td>
</tr>
<tr>
<td>${U_2^{F1}, U_3^{F1}} &amp; I_2$</td>
<td>$1 - \lambda + \lambda \pi - \lambda \pi \left( \frac{1 - \lambda}{2} + \lambda \pi \right)$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda + \frac{1 - \lambda \pi}{2} \right) \theta_1^I$</td>
</tr>
<tr>
<td>${U_1^{F1}, U_2^{F1}} &amp; I_3$</td>
<td>$(1 - \lambda + \lambda \pi) \left( 1 - \lambda + \lambda \pi \right)$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda + \lambda \pi \right) \theta_1^I$</td>
</tr>
<tr>
<td>$U_2^{F1} &amp; I_1$</td>
<td>$1 - \lambda + \lambda \pi$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda + \lambda \pi \right) \theta_1^I$</td>
</tr>
<tr>
<td>$U_2^{F1} &amp; I_2$</td>
<td>$1 - \lambda + \lambda \pi - \lambda \pi \left( \frac{1 - \lambda}{2} + \lambda \pi \right)$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda + \lambda \pi \right) \theta_1^I$</td>
</tr>
<tr>
<td>$U_3^{F1} &amp; I_1$</td>
<td>$1 - \lambda + \lambda \pi$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda + \lambda \pi \right) \theta_1^I$</td>
</tr>
<tr>
<td>$U_3^{F1} &amp; I_2$</td>
<td>$1 - \lambda + \lambda \pi - \lambda \pi \left( \frac{1 - \lambda}{2} + \lambda \pi \right)$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda + \lambda \pi \right) \theta_1^I$</td>
</tr>
<tr>
<td>$U_3^{F1} &amp; I_3$</td>
<td>$(1 - \lambda + \lambda \pi) \left( 1 - \lambda + \lambda \pi \right)$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda + \lambda \pi \right) \theta_1^I$</td>
</tr>
<tr>
<td>$U_4^{F1} &amp; I_1$</td>
<td>$1 - \lambda + \lambda \pi \left( 1 + \frac{\lambda(1 - \pi)}{2} \right)$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda + \lambda \pi \right) \theta_1^I$</td>
</tr>
<tr>
<td>$U_4^{F1} &amp; I_2$</td>
<td>$(1 - \lambda + \lambda \pi) \left( 1 + \frac{\lambda(1 - \pi)}{2} \right) - \lambda \pi \left( 1 - \lambda + \lambda \pi \right)$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda + \lambda \pi \right) \theta_1^I$</td>
</tr>
<tr>
<td>$U_4^{F1} &amp; I_3$</td>
<td>$(1 - \lambda + \lambda \pi) \left( 1 + \frac{\lambda(1 - \pi)}{2} - \lambda \pi \right)$</td>
<td>$\lambda^2 (1 - \pi)^2 \theta_1^U + \lambda \pi \left( \lambda + \lambda \pi \right) \theta_1^I$</td>
</tr>
<tr>
<td>Cases</td>
<td>$\mathbb{E}<em>0 \left( V</em>{E_X,2}^{\xi_U^I} \right)$</td>
<td>$\mathbb{E}<em>0 \left( V</em>{D_P,2}^{\xi_U^I} \right)$</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-----------------------------------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>${ U_1^{\xi_U^I}, U_2^{\xi_U^I} } &amp; I_1$</td>
<td>$1 - \lambda + \lambda \pi$</td>
<td>$\frac{1}{2} \lambda (1 - \pi) \theta_U^I$</td>
</tr>
<tr>
<td>${ U_1^{\xi_U^I}, U_3^{\xi_U^I} } &amp; I_2$</td>
<td>$1 - \lambda + \frac{\lambda \pi}{2} (\lambda + 1)$</td>
<td>$\frac{(1 - \lambda)}{2} \lambda \pi \theta_U^I + \frac{\lambda (1 - \pi)}{2} \theta_U^I$</td>
</tr>
<tr>
<td>${ U_1^{\xi_U^I}, U_3^{\xi_U^I} } &amp; { I_3, I_4 }$</td>
<td>$1 - \lambda + \lambda^2 \pi$</td>
<td>$(1 - \lambda) \lambda \pi \theta_U^I + \frac{1}{2} (1 - \pi) \theta_U^I$</td>
</tr>
<tr>
<td>${ U_1^{\xi_U^I}, U_3^{\xi_U^I} } &amp; I_5$</td>
<td>$1 - \lambda + \frac{\lambda \pi}{2} (1 - \pi)$</td>
<td>$\frac{1}{2} \lambda \pi (2 - \lambda + \lambda \pi) \theta_U^I + \frac{\lambda (1 - \pi)}{2} \theta_U^I$</td>
</tr>
<tr>
<td>${ U_1^{\xi_U^I}, U_3^{\xi_U^I} } &amp; I_6$</td>
<td>$1 - \lambda$</td>
<td>$\lambda \pi \theta_U^I + \frac{\lambda (1 - \pi)}{2} \theta_U^I$</td>
</tr>
<tr>
<td>$U_2^{\xi_U^I} &amp; I_1$</td>
<td>$1 - \lambda + \lambda \pi$</td>
<td>$(1 - \lambda) \lambda \pi \theta_U^I + \frac{1}{2} (1 - \lambda) \lambda (1 - \pi) \theta_U^I$</td>
</tr>
<tr>
<td>$U_2^{\xi_U^I} &amp; I_2$</td>
<td>$1 - \lambda + \frac{\lambda \pi}{2} (1 + \lambda)$</td>
<td>$(1 - \lambda) \left( \frac{\lambda \pi}{2} \theta_U^I + \frac{\lambda (1 - \pi)}{2} \theta_U^I \right)$</td>
</tr>
<tr>
<td>$U_2^{\xi_U^I} &amp; { I_3, I_4 }$</td>
<td>$1 - \lambda + \lambda^2 \pi$</td>
<td>$(1 - \lambda) \left( \lambda \pi \theta_U^I + \frac{\lambda (1 - \pi)}{2} \theta_U^I \right)$</td>
</tr>
<tr>
<td>$U_2^{\xi_U^I} &amp; I_5$</td>
<td>$1 - \lambda + \frac{\lambda \pi}{2} (1 - \pi)$</td>
<td>$\frac{\lambda \pi}{2} (2 - \lambda + \lambda \pi) \theta_U^I + \frac{(1 - \lambda) \lambda (1 - \pi)}{2} \theta_U^I$</td>
</tr>
<tr>
<td>$U_2^{\xi_U^I} &amp; I_6$</td>
<td>$1 - \lambda$</td>
<td>$(1 - \lambda) \left( \lambda \pi \theta_U^I + \frac{\lambda (1 - \pi)}{2} \theta_U^I \right)$</td>
</tr>
<tr>
<td>$U_4^{\xi_U^I} &amp; I_1$</td>
<td>$1 - \lambda + \frac{1}{2} (\lambda + (2 - \lambda) \pi)$</td>
<td>$(1 - \lambda) \left( \frac{\lambda \pi}{2} \theta_U^I + \frac{\lambda (1 - \pi)}{2} \theta_U^I \right)$</td>
</tr>
<tr>
<td>$U_4^{\xi_U^I} &amp; I_2$</td>
<td>$1 - \lambda + \frac{1}{2} (\lambda + \pi)$</td>
<td>$(1 - \lambda) \left( \frac{\lambda \pi}{2} \theta_U^I + \frac{\lambda (1 - \pi)}{2} \theta_U^I \right)$</td>
</tr>
<tr>
<td>$U_4^{\xi_U^I} &amp; { I_3, I_4 }$</td>
<td>$1 - \lambda + \lambda^2 \pi (1 + \pi)$</td>
<td>$(1 - \lambda) \left( \lambda \pi \theta_U^I + \frac{\lambda (1 - \pi)}{2} \theta_U^I \right)$</td>
</tr>
</tbody>
</table>
We find that the lowest ex-ante expected trading volume without the $DP$ is equal to the highest ex-ante expected volume with the $DP$. Therefore, $\mathbb{E}_0 \left( V_{EX,2}^{E_D} \right) \geq \mathbb{E}_0 \left( V_{EX,2}^{E_N} \right)$. This inequality might not be satisfied when we compare the total expected volume at $t = 2$, as shown in the following table:
<table>
<thead>
<tr>
<th>Cases</th>
<th>$E_0 \left( V_{T,2}^{\mathcal{E}^D} \right) - E_0 \left( V_{T,2}^{\mathcal{E}^{ND}} \right)$ if $X_{2,ND}^2 &gt; \frac{k_2}{\kappa}$</th>
<th>$E_0 \left( V_{T,2}^{\mathcal{E}^D} \right) - E_0 \left( V_{T,2}^{\mathcal{E}^{ND}} \right)$ if $X_{2,ND}^2 \leq \frac{k_2}{\kappa}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_1^{E_3^D} &amp; I_1$</td>
<td>$(1 - \lambda + \lambda \pi) \frac{\lambda(1-\pi)}{2} \theta^U_J$</td>
<td></td>
</tr>
<tr>
<td>$U_1^{E_3^D} &amp; I_2$</td>
<td>$(1 - \lambda + \lambda \pi) \frac{\lambda(1-\pi)}{2} \theta^U_J + \frac{\lambda \pi}{2} (1 - \lambda + 2\lambda \pi) (\theta^U_J - 1)$</td>
<td>$(1 - \lambda + \lambda \pi) \frac{\lambda(1-\pi)}{2} \theta^U_J + \frac{\lambda \pi}{2} (1 - \lambda + 2\lambda \pi) (\theta^U_J - 1)$</td>
</tr>
<tr>
<td>$U_1^{E_3^D} &amp; I_3$</td>
<td>$(1 - \lambda + \lambda \pi) \frac{\lambda(1-\pi)}{2} \theta^U_J + \lambda \pi (1 - \lambda + \lambda \pi) (\theta^U_J - 1)$</td>
<td></td>
</tr>
<tr>
<td>$U_2^{E_3^D} &amp; I_1$</td>
<td></td>
<td>$0$</td>
</tr>
<tr>
<td>$U_2^{E_3^D} &amp; I_2$</td>
<td></td>
<td>$\frac{\lambda \pi}{2} (1 - \lambda + 2\lambda \pi) (\theta^U_J - 1)$</td>
</tr>
<tr>
<td>$U_2^{E_3^D} &amp; I_3$</td>
<td></td>
<td>$\lambda \pi (1 - \lambda + \lambda \pi) (\theta^U_J - 1)$</td>
</tr>
<tr>
<td>$U_3^{E_3^D} &amp; I_1$</td>
<td>$\frac{\lambda(1-\pi)}{2} (1 - \lambda + \lambda \pi) (\theta^U_J - 1)$</td>
<td>$\frac{\lambda(1-\pi)}{2} (1 - \lambda + \lambda \pi) \theta^U_J$</td>
</tr>
<tr>
<td>$U_3^{E_3^D} &amp; I_2$</td>
<td>$\frac{\lambda(1-\pi)}{2} (1 - \lambda + \lambda \pi) (\theta^U_J - 1) + \frac{\lambda \pi}{2} (1 - \lambda + 2\lambda \pi) (\theta^U_J - 1)$</td>
<td>$\frac{\lambda(1-\pi)}{2} (1 - \lambda + \lambda \pi) \theta^U_J + \frac{\lambda \pi}{2} (1 - \lambda + 2\lambda \pi) (\theta^U_J - 1)$</td>
</tr>
<tr>
<td>$U_3^{E_3^D} &amp; I_3$</td>
<td>$\frac{\lambda(1-\pi)}{2} (1 - \lambda + \lambda \pi) (\theta^U_J - 1) + \lambda \pi (1 - \lambda + \lambda \pi) (\theta^U_J - 1)$</td>
<td>$\frac{\lambda(1-\pi)}{2} (1 - \lambda + \lambda \pi) \theta^U_J + \lambda \pi (1 - \lambda + \lambda \pi) (\theta^U_J - 1)$</td>
</tr>
<tr>
<td>$U_4^{E_3^D} &amp; I_1$</td>
<td></td>
<td>$0$</td>
</tr>
<tr>
<td>$U_4^{E_3^D} &amp; I_2$</td>
<td>$\frac{\lambda \pi}{2} (1 - \lambda + 2\lambda \pi) (\theta^U_J - 1)$</td>
<td></td>
</tr>
<tr>
<td>$U_4^{E_3^D} &amp; I_3$</td>
<td>$\lambda \pi (1 - \lambda + \lambda \pi) (\theta^U_J - 1)$</td>
<td></td>
</tr>
</tbody>
</table>

- $\mathcal{E}_3^{ND}$ and $\mathcal{E}_3^{D}$

**Case 1:** $Y_{3,ND}^3 = \pi > \frac{k_1}{\kappa}$. The highest ex-ante expected volume at $t = 2$ without the $DP$ is $E_0 \left( V_{EX,2}^{\mathcal{E}^{ND}} \right) = 1 - \lambda + \lambda \pi + \frac{\lambda^2}{2} (1 - \pi)$. Hence, $E_0 \left( V_{EX,2}^{\mathcal{E}^{ND}} \right) \geq E_0 \left( V_{EX,2}^{\mathcal{E}^D} \right)$.

**Case 2:** $Y_{3,ND}^3 = \pi \leq \frac{k_1}{\kappa}$. The highest ex-ante expected volume at $t = 2$ without the $DP$ is $E_0 \left( V_{EX,2}^{\mathcal{E}^{ND}} \right) = 1 - \lambda + \lambda \pi$. Hence, in this case we also have that $E_0 \left( V_{EX,2}^{\mathcal{E}^{ND}} \right) \geq E_0 \left( V_{EX,2}^{\mathcal{E}^D} \right)$.

With regards to expected total volume, we find that this inequality might not be satisfied when we compare the total expected volume at $t = 2$, as shown in the following table:
$\mathcal{E}_4^{\text{ND}}$ and $\mathcal{E}_4^{\text{D}}$

Using the information of the relevant tables, we find that the lowest ex-ante expected volume with no dark is equal to the highest ex-ante expected volume with the dark. Therefore, $E_0\left(V^{\mathcal{E}_4^{\text{ND},2}}\right) \geq E_0\left(V^{\mathcal{E}_4^{\text{D},2}}\right)$. However, this inequality might not be satisfied when we compare the total ex-ante expected volume at $t = 2$, as indicated in the following table:

<table>
<thead>
<tr>
<th>Cases</th>
<th>$E_0\left(V^{\mathcal{E}_4^{\text{D},2}}\right)$</th>
<th>$E_0\left(V^{\mathcal{E}_4^{\text{ND},2}}\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_1^{2T_u} &amp; I_1$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U$</td>
</tr>
<tr>
<td>$U_1^{2T_u} &amp; I_2$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
</tr>
<tr>
<td>$U_1^{2T_u} &amp; I_3$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
</tr>
<tr>
<td>$U_1^{2T_u} &amp; I_4$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
</tr>
<tr>
<td>$U_2^{2T_u} &amp; I_1$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U$</td>
</tr>
<tr>
<td>$U_2^{2T_u} &amp; I_2$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
</tr>
<tr>
<td>$U_2^{2T_u} &amp; I_3$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
</tr>
<tr>
<td>$U_2^{2T_u} &amp; I_4$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
</tr>
<tr>
<td>$U_3^{2T_u} &amp; I_5$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U$</td>
</tr>
<tr>
<td>$U_3^{2T_u} &amp; I_6$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
</tr>
<tr>
<td>$U_3^{2T_u} &amp; I_7$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
<td>$\frac{\lambda(1-\lambda)(1-\rho)}{2}$ $\theta_1^U + \frac{\lambda\pi(1-\rho)}{2}$ $\theta_1^U$</td>
</tr>
</tbody>
</table>

81
\( E_1^{ND} \) and \( E_5^D \)

Using the information of the relevant tables, we notice that the lowest ex-ante expected volume corresponding to the equilibrium without the \( DP(E_1^{ND}) \) is equal to the highest ex-ante expected volume corresponding to the equilibrium with the dark \( E_5^D \). Therefore, \( \mathbb{E}_0 \left( V_{t,2}^{E_1^{ND}} \right) \geq \mathbb{E}_0 \left( V_{t,2}^{E_5^D} \right) \).

This inequality might not be satisfied when we compare the total ex-ante expected volume at \( t = 2 \):

\[
\begin{array}{|c|c|}
\hline
\text{Cases} & \mathbb{E}_0 \left( V_{t,2}^{E_1^{ND}} \right) - \mathbb{E}_0 \left( V_{t,2}^{E_5^D} \right) \\
\hline
I_1 & \lambda \pi (1 - \theta_1^0) / 2 + \lambda (1 - \pi) (1 - \lambda) \theta_2^U \\
I_2 & \lambda \pi (1 - \theta_1^0) + \lambda (1 - \pi) (1 - \lambda) \theta_2^U + \lambda \pi (1 - \lambda) (\theta_2^0 - 1) \\
I_3 & \lambda \pi (1 - \theta_1^0) + \lambda (1 - \pi) (1 - \lambda) \theta_2^U + \lambda (1 - \lambda) (\theta_2^0 - 1) \\
I_4 & \lambda \pi (1 - \theta_1^0) + \lambda (1 - \pi) (1 - \lambda) \theta_2^U + \lambda \pi (1 - \lambda) (\theta_2^0 - 1) \\
I_5 & \lambda \pi (1 - \theta_1^0) + \lambda (1 - \pi) (1 - \lambda) \theta_2^U + \lambda \pi (1 - \lambda) (\theta_2^0 - 1) \\
I_6 & \lambda \pi (1 - \theta_1^0) + \lambda (1 - \pi) (1 - \lambda) \theta_2^U + \lambda \pi (1 - \lambda) (\theta_2^0 - 1) \\
\hline
\end{array}
\]

\( \mathbb{E}_0 \left( V_{t,2}^{E_1^{ND}} \right) = 1 - \lambda + \lambda \pi \)

Using the information of the relevant tables, notice that we obtain ambiguous results when we compare the ex-ante expected volume in the exchange. This ambiguity is also satisfied when we compare the total ex-ante expected volume in the second trading period:

\( E_2^{ND} \) and \( E_5^D \)

The migration from \( E_2^{ND} \) to \( E_5^D \) has only sense in the case \( I_6 \). Using the relevant tables, we conclude that \( \mathbb{E}_0 \left( V_{t,2}^{E_2^{ND}} \right) \geq \mathbb{E}_0 \left( V_{t,2}^{E_5^D} \right) \). With regards to trade creation, we obtain ambiguous results as indicated in the following table:

\[
\begin{array}{|c|c|}
\hline
\text{Cases} & \mathbb{E}_0 \left( V_{t,2}^{E_2^{ND}} \right) - \mathbb{E}_0 \left( V_{t,2}^{E_2^{ND}} \right) \text{ if } X^{2,ND} > \frac{\kappa_2}{K} \\
\hline
I_6 & -\frac{\lambda (1 - \pi)}{2} (1 - \lambda + \lambda \pi) + \lambda (1 - \pi) (1 - \lambda) \theta_2^U + \lambda \pi (\theta_2^0 - 1) + \lambda \pi (1 - \theta_1^0) \\
\hline
\end{array}
\]

\( E_3^{ND} \) and \( E_5^D \)

Using the relevant tables, we obtain that the lowest ex-ante expected volume with no dark \( E_3^{ND} \) is equal to the highest ex-ante expected volume with the dark \( E_5^D \). Therefore, \( \mathbb{E}_0 \left( V_{t,2}^{E_3^{ND}} \right) \geq \mathbb{E}_0 \left( V_{t,2}^{E_5^D} \right) \).
Using the information of the relevant tables, notice that we obtain ambiguous results when we compare the ex-ante expected volume in the exchange. With regards to trade creation, we find ambiguous results as shown in the following table:

<table>
<thead>
<tr>
<th>Cases</th>
<th>( E_0 \left( V_{E,2}^{EX} \right) - E_0 \left( V_{E,2}^{EX,2} \right) ) if ( \pi &gt; \frac{k_1}{\kappa} )</th>
<th>( E_0 \left( V_{E,2}^{EX} \right) - E_0 \left( V_{E,2}^{EX,2} \right) ) if ( \pi \leq \frac{k_1}{\kappa} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_1 )</td>
<td>(-\lambda(1-\pi) + \frac{\lambda(1-\pi)(1-\lambda\pi)}{2} \theta_2^0 + \lambda\pi(1-\theta_1^0))</td>
<td>(-\lambda(1-\pi) + \frac{\lambda(1-\pi)(1-\lambda\pi)}{2} \theta_2^0 + \lambda\pi(1-\theta_1^0))</td>
</tr>
<tr>
<td>( I_2 )</td>
<td>(-\lambda(1-\pi) + \frac{\lambda(1-\pi)(1-\lambda\pi)}{2} \theta_2^0 + \lambda\pi(1-\theta_1^0))</td>
<td>(-\lambda(1-\pi) + \frac{\lambda(1-\pi)(1-\lambda\pi)}{2} \theta_2^0 + \lambda\pi(1-\theta_1^0))</td>
</tr>
<tr>
<td>( I_3 )</td>
<td>(-\lambda(1-\pi) + \frac{\lambda(1-\pi)(1-\lambda\pi)}{2} \theta_2^0 + \lambda\pi(1-\theta_1^0))</td>
<td>(-\lambda(1-\pi) + \frac{\lambda(1-\pi)(1-\lambda\pi)}{2} \theta_2^0 + \lambda\pi(1-\theta_1^0))</td>
</tr>
<tr>
<td>( I_4 )</td>
<td>(-\lambda(1-\pi) + \frac{\lambda(1-\pi)(1-\lambda\pi)}{2} \theta_2^0 + \lambda\pi(1-\theta_1^0))</td>
<td>(-\lambda(1-\pi) + \frac{\lambda(1-\pi)(1-\lambda\pi)}{2} \theta_2^0 + \lambda\pi(1-\theta_1^0))</td>
</tr>
<tr>
<td>( I_5 )</td>
<td>(-\lambda(1-\pi) + \frac{\lambda(1-\pi)(1-\lambda\pi)}{2} \theta_2^0 + \lambda\pi(1-\theta_1^0))</td>
<td>(-\lambda(1-\pi) + \frac{\lambda(1-\pi)(1-\lambda\pi)}{2} \theta_2^0 + \lambda\pi(1-\theta_1^0))</td>
</tr>
<tr>
<td>( I_6 )</td>
<td>(-\lambda(1-\pi) + \frac{\lambda(1-\pi)(1-\lambda\pi)}{2} \theta_2^0 + \lambda\pi(1-\theta_1^0))</td>
<td>(-\lambda(1-\pi) + \frac{\lambda(1-\pi)(1-\lambda\pi)}{2} \theta_2^0 + \lambda\pi(1-\theta_1^0))</td>
</tr>
</tbody>
</table>

- \( \mathcal{E}_4^{ND} \) and \( \mathcal{E}_5^D \)

The migration from \( \mathcal{E}_4^{ND} \) to \( \mathcal{E}_5^D \) has only sense in the case \( I_6 \). Using the relevant tables, we conclude that \( E_0 \left( V_{E,2}^{EX} \right) > E_0 \left( V_{E,2}^{EX,2} \right) \).

With regards to trade creation, we obtain ambiguous results as indicated in the following table:

<table>
<thead>
<tr>
<th>Cases</th>
<th>( E_0 \left( V_{E,2}^{EX} \right) - E_0 \left( V_{E,2}^{EX,2} \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_6 )</td>
<td>(-\lambda(1-\pi) + \frac{\lambda(1-\pi)(1-\lambda\pi)}{2} \theta_2^0 + \lambda\pi(1-\theta_1^0))</td>
</tr>
</tbody>
</table>

- \( \mathcal{E}_2^{ND} \) and \( \mathcal{E}_6^D \)

Using the relevant tables, we notice that the lowest ex-ante expected volume corresponding to the equilibrium without the \( DP \) (\( \mathcal{E}_2^{ND} \)) is equal to the highest ex-ante expected volume corresponding to the equilibrium with the \( DP \) (\( \mathcal{E}_6^D \)). Therefore, \( E_0 \left( V_{E,2}^{EX} \right) \geq E_0 \left( V_{E,2}^{EX,2} \right) \). With regards to trade creation, we find ambiguous results as shown in the following table:5

5In the following tables we omit the case \( I_6 \) because in this case it requires \( k_1 = 1 \).
Using the information of the relevant tables, notice that we obtain ambiguous results when we compare the ex-ante expected volume in the exchange. With regards to trade creation, we find ambiguous results as shown in the following table.\(^6\)

<table>
<thead>
<tr>
<th>Cases</th>
<th>(\mathbb{E}<em>0\left(V</em>{E_{X,2}}^{E_0}\right)) if (X^{2,ND} &gt; \frac{k_2}{\kappa})</th>
<th>(\mathbb{E}<em>0\left(V</em>{D_{P,2}}^{E_0}\right))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_1)</td>
<td>(-\frac{\lambda(1-\pi)}{2}(1-\lambda+\pi) + \frac{(1-\lambda)\lambda(1-\pi)}{2}\theta_2^U + \lambda\pi(1-\theta_1^I))</td>
<td>(\frac{(1-\lambda)\lambda(1-\pi)}{2}\theta_2^U + \lambda\pi(1-\theta_1^I))</td>
</tr>
<tr>
<td>(I_2)</td>
<td>(-\frac{\lambda(1-\pi)}{2}(1-\lambda+\pi) + \frac{(1-\lambda)\lambda(1-\pi)}{2}\theta_2^U + \frac{\lambda\pi(1-\lambda)}{(1-\lambda)}(\theta_2^U - 1) + \lambda\pi(1-\theta_1^I))</td>
<td>(\frac{(1-\lambda)(1-\lambda)}{2}\theta_2^U + \frac{\lambda\pi(1-\lambda)}{(1-\lambda)}(\theta_2^U - 1) + \lambda\pi(1-\theta_1^I))</td>
</tr>
<tr>
<td>(I_3)</td>
<td>(-\frac{\lambda(1-\pi)}{2}(1-\lambda+\pi) + \frac{(1-\lambda)\lambda(1-\pi)}{2}\theta_2^U + \frac{\lambda\pi(1-\lambda)}{(1-\lambda)}(\theta_2^U - 1) + \lambda\pi(1-\theta_1^I))</td>
<td>(\frac{(1-\lambda)(1-\lambda)}{2}\theta_2^U + \frac{\lambda\pi(1-\lambda)}{(1-\lambda)}(\theta_2^U - 1) + \lambda\pi(1-\theta_1^I))</td>
</tr>
<tr>
<td>({I_4, I_5, I_6})</td>
<td>(-\frac{\lambda(1-\pi)}{2}(1-\lambda+\pi) + \frac{(1-\lambda)\lambda(1-\pi)}{2}\theta_2^U + \lambda\pi(1-\theta_1^I))</td>
<td>(\frac{(1-\lambda)\lambda(1-\pi)}{2}\theta_2^U + \lambda\pi(1-\theta_1^I))</td>
</tr>
</tbody>
</table>

\(\bullet \ E_4^{ND}\) and \(E_6^D\)

Using the relevant tables, we notice that the ex-ante expected volume corresponding to the equilibrium without the \(DP\) (\(E_4^{ND}\)) is larger than the highest ex-ante expected volume corresponding to the equilibrium with the \(DP\) (\(E_6^D\)). Therefore, \(\mathbb{E}_0\left(V_{E_{X,2}}^{E_0}\right) \geq \mathbb{E}_0\left(V_{E_{X,2}}^{E_0}\right)\). With regards to trade creation, we find ambiguous results as shown in the following table:

<table>
<thead>
<tr>
<th>Cases(E_6^D, t = 2)</th>
<th>(\mathbb{E}<em>0\left(V</em>{E_{X,2}}^{E_0}\right) - \mathbb{E}<em>0\left(V</em>{E_{X,2}}^{E_0}\right))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_1)</td>
<td>(-\frac{\lambda^2(1-\pi)}{2} + \frac{(1-\lambda)\lambda(1-\pi)}{2}\theta_2^U)</td>
</tr>
<tr>
<td>(I_2)</td>
<td>(-\frac{\lambda^2(1-\pi)}{2} + \frac{(1-\lambda)\lambda(1-\pi)}{2}\theta_2^U + \frac{(1-\lambda)\lambda\pi}{(1-\lambda)}(\theta_2^U - 1))</td>
</tr>
<tr>
<td>(I_3)</td>
<td>(-\frac{\lambda^2(1-\pi)}{2} + \frac{(1-\lambda)\lambda(1-\pi)}{2}\theta_2^U + (1-\lambda)\lambda\pi(\theta_2^U - 1))</td>
</tr>
<tr>
<td>({I_4, I_5})</td>
<td>(-\frac{\lambda^2(1-\pi)}{2} + \frac{(1-\lambda)\lambda(1-\pi)}{2}\theta_2^U + \lambda\pi(\theta_2^U - 1))</td>
</tr>
</tbody>
</table>

To compare the volume in the exchange we notice that

\(6\)In the following tables we omit the case \(I_6\) because in this case it requires \(k_1 = 1\).
Using the information of the relevant tables, notice that we obtain ambiguous results when we compare the ex-ante expected volume in the exchange. With regards to trade creation, we find ambiguous results as shown in the following table:

<table>
<thead>
<tr>
<th>Cases $\varepsilon^{D}_{6}, t = 2$</th>
<th>$\mathbb{E}<em>{0}\left( V</em>{EX,2}^{F_{i}} \right) - \mathbb{E}<em>{0}\left( V</em>{ND,2}^{F_{j}} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1$</td>
<td>$-\frac{\lambda^{2} \pi (1-\pi)}{2} + (1-\lambda) \lambda (1-\pi) \theta^{U}<em>{2} + \lambda \pi (1-\theta^{I}</em>{1})$</td>
</tr>
<tr>
<td>$I_2$</td>
<td>$-\frac{\lambda^{2} \pi (1-\pi)}{2} + (1-\lambda) \lambda (1-\pi) \theta^{U}<em>{2} + (1-\lambda) \lambda \pi (\theta^{I}</em>{2} - 1) + \lambda \pi (1-\theta^{I}_{1})$</td>
</tr>
<tr>
<td>$I_3$</td>
<td>$-\frac{\lambda^{2} \pi (1-\pi)}{2} + (1-\lambda) \lambda (1-\pi) \theta^{U}<em>{2} + (1-\lambda) \lambda \pi (\theta^{I}</em>{2} - 1) + \lambda \pi (1-\theta^{I}_{1})$</td>
</tr>
<tr>
<td>${I_4, I_5, I_6}$</td>
<td>$-\frac{\lambda^{2} \pi (1-\pi)}{2} + (1-\lambda) \lambda (1-\pi) \theta^{U}<em>{2} + \lambda \pi (\theta^{I}</em>{2} - 1) + \lambda \pi (1-\theta^{I}_{1})$</td>
</tr>
</tbody>
</table>

To sum up, with regards to volume comparison in the exchange at $t = 2$, we obtain that:

<table>
<thead>
<tr>
<th>$V_{EX,2}$</th>
<th>$\varepsilon^{D}_{1}$</th>
<th>$\varepsilon^{D}_{2}$</th>
<th>$\varepsilon^{D}_{3}$</th>
<th>$\varepsilon^{D}_{4}$</th>
<th>$\varepsilon^{D}_{5}$</th>
<th>$\varepsilon^{D}_{6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon^{ND}_{1}$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
</tr>
<tr>
<td>$\varepsilon^{ND}_{2}$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
</tr>
<tr>
<td>$\varepsilon^{ND}_{3}$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
</tr>
<tr>
<td>$\varepsilon^{ND}_{4}$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
</tr>
</tbody>
</table>

while the results related to total volume are ambiguous.

To sum up, with regards to volume comparison in the exchange at $t = 2$, we obtain that:

<table>
<thead>
<tr>
<th>$V_{EX,2}$</th>
<th>$\varepsilon^{D}_{1}$</th>
<th>$\varepsilon^{D}_{2}$</th>
<th>$\varepsilon^{D}_{3}$</th>
<th>$\varepsilon^{D}_{4}$</th>
<th>$\varepsilon^{D}_{5}$</th>
<th>$\varepsilon^{D}_{6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon^{ND}_{1}$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
</tr>
<tr>
<td>$\varepsilon^{ND}_{2}$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
</tr>
<tr>
<td>$\varepsilon^{ND}_{3}$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
</tr>
<tr>
<td>$\varepsilon^{ND}_{4}$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
<td>$\geq$</td>
</tr>
</tbody>
</table>

while the results related to total volume are ambiguous.
Proof of Proposition 6. We first summarize the expected profits for rational traders at \( t = 1 \) in the single-venue market:

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Expected Profits of an Informed Trader at ( t = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{E}_5^D )</td>
<td>( \theta_1^I (\kappa - k_1) \tau + (1 - \theta_2^I) \delta (\kappa - k_1 - (k_2 - k_1)(\lambda \pi + 1 - \lambda) - \lambda \pi \kappa) \tau ) if ( \theta_2^I \leq \frac{\kappa - k_1}{\kappa} )</td>
</tr>
<tr>
<td>( \mathcal{E}_6^D )</td>
<td>( \theta_1^I (\kappa - k_1 - (k_2 - k_1)(\lambda \pi + 1 - \lambda)) \tau ) if ( \theta_2^I \leq \frac{\kappa - k_1}{\kappa} )</td>
</tr>
</tbody>
</table>

and

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Expected Profits of an Uninformed Trader at ( t = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{E}_5^D )</td>
<td>( \frac{\delta}{2} (\lambda \pi + 1 - \lambda)(k_1 - 1) \tau ) if ( \theta_2^I \leq \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}} )</td>
</tr>
<tr>
<td>( \mathcal{E}_6^D )</td>
<td>0</td>
</tr>
</tbody>
</table>

When traders have access to the \( DP \), the unconditional expected profits of investors for the first four equilibria coincide with the corresponding ones given in the previous table. For the two remaining equilibria, we have the following unconditional expected profits:

The following tables show how the unconditional expected profits of rational traders change due to the introduction of the \( DP \) at \( t = 1 \):

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Expected Profits of an Informed Trader at ( t = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{E}_1^D )</td>
<td>=</td>
</tr>
<tr>
<td>( \mathcal{E}_2^D )</td>
<td>=</td>
</tr>
<tr>
<td>( \mathcal{E}_3^D )</td>
<td>=</td>
</tr>
<tr>
<td>( \mathcal{E}_4^D )</td>
<td>=</td>
</tr>
</tbody>
</table>

and

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Expected Profits of an Uninformed Trader at ( t = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{E}_1^D )</td>
<td>=</td>
</tr>
<tr>
<td>( \mathcal{E}_2^D )</td>
<td>=</td>
</tr>
<tr>
<td>( \mathcal{E}_3^D )</td>
<td>=</td>
</tr>
<tr>
<td>( \mathcal{E}_4^D )</td>
<td>=</td>
</tr>
</tbody>
</table>

and

The following tables show how the unconditional expected profits of rational traders change due to the introduction of the \( DP \) at \( t = 1 \):

<table>
<thead>
<tr>
<th>Informed Trader</th>
<th>( \mathcal{E}_1^D )</th>
<th>( \mathcal{E}_2^D )</th>
<th>( \mathcal{E}_3^D )</th>
<th>( \mathcal{E}_4^D )</th>
<th>( \mathcal{E}_5^D )</th>
<th>( \mathcal{E}_6^D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{E}_1^{ND} )</td>
<td>=</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
</tr>
</tbody>
</table>

and

<table>
<thead>
<tr>
<th>Uninformed Trader</th>
<th>( \mathcal{E}_1^D )</th>
<th>( \mathcal{E}_2^D )</th>
<th>( \mathcal{E}_3^D )</th>
<th>( \mathcal{E}_4^D )</th>
<th>( \mathcal{E}_5^D )</th>
<th>( \mathcal{E}_6^D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{E}_1^{ND} )</td>
<td>=</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
<td>&lt;</td>
</tr>
</tbody>
</table>
Next, we compare the expected profits at \( t = 2 \) of the two-venue market and the single-venue market. The following tables display the expected profits at \( t = 2 \) without and with the \( DP \) in each of the possible equilibria. The calculations supporting the following tables for expected profits without the \( DP \) and with the \( DP \) can be obtained on request from the authors. Again, in what follows, \( I_1 \) - \( I_6 \) and \( U^{EP}_j, \ i = 1, ..., 6 \), correspond to the cases referred to in the analysis performed to derive the equilibria in the two-venue market given in Internet Appendix II.

The expected trading profits at \( t = 2 \) without the \( DP \) in each equilibria are the following:

<table>
<thead>
<tr>
<th>Expected Profits at ( t = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Informed Trader</td>
</tr>
<tr>
<td>( \mathcal{E}^{ND}_1 ) \quad \frac{2(\kappa-k_2)+\lambda(1-\pi)+(k_2-k_1)(-2\lambda\pi+\lambda+1)}{2} \tau \quad \lambda\pi(k_2-k_1) \tau \quad \frac{\lambda\pi-\lambda\pi\kappa}{2} \tau</td>
</tr>
<tr>
<td>Uninformed Trader</td>
</tr>
<tr>
<td>( \mathcal{E}^{ND}_2 ) \quad \frac{2(\kappa-k_2)+(k_2-k_1)(-2\lambda\pi+\lambda+1)}{2} \tau \quad \lambda\pi \quad \lambda\pi \kappa \frac{2}{\kappa} \tau</td>
</tr>
</tbody>
</table>

Similarly, the expected trading profits at \( t = 2 \) with the \( DP \) in each equilibria are the following:

<table>
<thead>
<tr>
<th>Expected Profits at ( t = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Informed trader</td>
</tr>
<tr>
<td>Case ( I_1 ) \quad \frac{2(\kappa-k_2)+\lambda(1-\pi)+(k_2-k_1)(-2\lambda\pi+\lambda+1)}{2} \tau \quad \frac{\lambda\pi-\lambda\pi\kappa}{2} \tau</td>
</tr>
<tr>
<td>Case ( I_2 ) \quad \theta_f^D \left( \lambda\pi+\frac{1-\lambda}{2} \right) \left( k_2-k_1 \right) \tau \quad \frac{\lambda\pi}{2} \tau</td>
</tr>
<tr>
<td>Case ( I_3 ) \quad \theta_f^D \left( \lambda\pi-\lambda\pi\kappa \right) \left( k_2-k_1 \right) \tau \quad \frac{\lambda\pi}{2} \tau</td>
</tr>
</tbody>
</table>

| Uninformed trader                |
| Case \( U^{EP}_1 \) \quad \frac{\lambda(1-\pi)}{2} \quad \frac{\lambda(1-\pi)}{2} \tau |
| Case \( U^{EP}_2 \) \quad \frac{\theta_f^D \left( 2\pi\kappa\lambda-(1-\lambda+\lambda\pi)\left(k_2-k_1\right)+\lambda(1-\pi) \right) \tau}{4} |
| Case \( U^{EP}_3 \) \quad \frac{\theta_f^D \left( 2\pi\kappa\lambda-(1-\lambda+\lambda\pi)\left(k_2-k_1\right)+\lambda(1-\pi) \right) \tau}{4} |

87
We then perform a pairwise comparison of the profits with and without \( DP \).

- \( \xi^{ND}_1 \) and \( \xi^D_1 \)

  Using the information of the relevant tables, we notice that
the expected profits of informed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{ND}_1 \) are equal to the lowest expected profits of informed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{D}_1 \) (the expected profits in the case \( I_1 \)). Therefore, we conclude that \( \mathbb{E}_0 \left( \Pi_{2,I}^{\mathcal{E}^{ND}_1} \right) \leq \mathbb{E}_0 \left( \Pi_{2,I}^{\mathcal{E}^{D}_1} \right) \).

- the expected profits of uninformed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{ND}_1 \) are null, while the expected profits of uninformed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{D}_1 \) are strictly positive in all the possible cases whenever \( \theta_U^I > 0 \). Therefore, we conclude that \( \mathbb{E}_0 \left( \Pi_{2,U}^{\mathcal{E}^{ND}_1} \right) < \mathbb{E}_0 \left( \Pi_{2,U}^{\mathcal{E}^{D}_1} \right) \).

• \( \mathcal{E}^{ND}_1 \) and \( \mathcal{E}^{D}_5 \)

Using the information of the relevant tables, we notice that

- the expected profits of informed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{ND}_1 \) are lower than the lowest expected profits of informed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{D}_1 \) (the expected profits in the case \( I_1 \)). Therefore, we conclude that \( \mathbb{E}_0 \left( \Pi_{2,I}^{\mathcal{E}^{D}_5} \right) < \mathbb{E}_0 \left( \Pi_{2,I}^{\mathcal{E}^{D}_1} \right) \).

- the expected profits of uninformed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{ND}_1 \) are null, while the expected profits of uninformed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{D}_1 \) are strictly positive whenever \( \theta_U^I > 0 \). Therefore, we conclude that \( \mathbb{E}_0 \left( \Pi_{2,U}^{\mathcal{E}^{ND}_1} \right) < \mathbb{E}_0 \left( \Pi_{2,U}^{\mathcal{E}^{D}_1} \right) \).

• \( \mathcal{E}^{ND}_2 \) and \( \mathcal{E}^{D}_5 \)

Using the information of the relevant tables, we notice that

- the expected profits of informed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{ND}_2 \) are equal to the lowest expected profits of informed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{D}_2 \) (the expected profits in the case \( I_1 \)). Therefore, we conclude that \( \mathbb{E}_0 \left( \Pi_{2,I}^{\mathcal{E}^{ND}_2} \right) \leq \mathbb{E}_0 \left( \Pi_{2,I}^{\mathcal{E}^{D}_2} \right) \).

- the expected profits of uninformed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{ND}_2 \) are lower or equal to the expected profits of uninformed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{D}_2 \), i.e., we conclude that \( \mathbb{E}_0 \left( \Pi_{2,U}^{\mathcal{E}^{ND}_2} \right) \leq \mathbb{E}_0 \left( \Pi_{2,U}^{\mathcal{E}^{D}_2} \right) \).

• \( \mathcal{E}^{ND}_2 \) and \( \mathcal{E}^{D}_5 \)

Using the information of the relevant tables, we notice that

- the expected profits of informed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{ND}_2 \) are lower than the lowest expected profits of informed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{D}_5 \) (the expected profits in the case \( I_1 \)). Therefore, in all the cases, but in particular the relevant one, \( I_6 \), we have that \( \mathbb{E}_0 \left( \Pi_{2,I}^{\mathcal{E}^{ND}_2} \right) < \mathbb{E}_0 \left( \Pi_{2,I}^{\mathcal{E}^{D}_5} \right) \).

- the comparison between expected profits of uninformed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{ND}_2 \) and the expected profits of uninformed traders at \( t = 2 \) corresponding to \( \mathcal{E}^{D}_5 \) is in general ambiguous. Direct computations yield that
Using the information of the relevant tables, we notice that

- the expected profits of informed traders at $t = 2$ corresponding to $\mathcal{E}_2^{ND}$ are lower than the lowest expected profits of informed traders at $t = 2$ corresponding to $\mathcal{E}_6^D$ (the expected profits in the case $I_1$). Therefore, we conclude that $E_0\left(\Pi_{2,U}^{\mathcal{E}_2^{ND}}\right) < E_0\left(\Pi_{2,U}^{\mathcal{E}_6^D}\right)$.

- the comparison between expected profits of uninformed traders at $t = 2$ corresponding to $\mathcal{E}_2^{ND}$ and the expected profits of uninformed traders at $t = 2$ corresponding to $\mathcal{E}_6^D$ is in general ambiguous. Direct computations yield that

$$E_0\left(\Pi_{2,U}^{\mathcal{E}_2^{ND}}\right) = E_0\left(\Pi_{2,U}^{\mathcal{E}_6^D}\right).$$

Using the information of the relevant tables, we notice that

- the expected profits of informed traders at $t = 2$ corresponding to $\mathcal{E}_3^{ND}$ are equal to the lowest expected profits of informed traders at $t = 2$ corresponding to $\mathcal{E}_4^D$ (the expected profits in the case $I_1$). Therefore, we conclude that $E_0\left(\Pi_{2,I}^{\mathcal{E}_3^{ND}}\right) = E_0\left(\Pi_{2,I}^{\mathcal{E}_4^D}\right)$.

- the expected profits of uninformed traders at $t = 2$ corresponding to $\mathcal{E}_3^{ND}$ are lower than the expected profits of uninformed traders at $t = 2$ corresponding to $\mathcal{E}_4^D$ in all the possible cases whenever $\theta_2^U > 0$. Therefore, we conclude that $E_0\left(\Pi_{2,U}^{\mathcal{E}_3^{ND}}\right) < E_0\left(\Pi_{2,U}^{\mathcal{E}_4^D}\right)$.

Using the information of the relevant tables, we notice that

- the expected profits of informed traders at $t = 2$ corresponding to $\mathcal{E}_3^{ND}$ are equal to the lowest expected profits of informed traders at $t = 2$ corresponding to $\mathcal{E}_5^D$ (the expected profits in the case $I_1$). Therefore, we conclude that $E_0\left(\Pi_{2,I}^{\mathcal{E}_3^{ND}}\right) = E_0\left(\Pi_{2,I}^{\mathcal{E}_5^D}\right)$.

- the comparison between expected profits of uninformed traders at $t = 2$ corresponding to $\mathcal{E}_3^{ND}$ and the expected profits of uninformed traders at $t = 2$ corresponding to $\mathcal{E}_5^D$ is in general ambiguous. Direct computations yield that

$$E_0\left(\Pi_{2,U}^{\mathcal{E}_3^{ND}}\right) = E_0\left(\Pi_{2,U}^{\mathcal{E}_5^D}\right).$$
### Comparison

<table>
<thead>
<tr>
<th>Condition</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi \leq \frac{k_1}{K}$ or $\pi &gt; \frac{k_1}{K}$ and $\theta^U_2 &gt; \frac{2\lambda(\pi_2 - k_1)}{(1-\lambda)(k_2-k_1) + \lambda(1-\pi)}$</td>
<td>$E_0 \left( \Pi^{ND}<em>{2,\Omega} \right) &lt; E_0 \left( \Pi^{D}</em>{2,\Omega} \right)$</td>
</tr>
<tr>
<td>$\pi &gt; \frac{k_1}{K}$ and $\theta^U_2 \leq \frac{2\lambda(\pi_2 - k_1)}{(1-\lambda)(k_2-k_1) + \lambda(1-\pi)}$</td>
<td>$E_0 \left( \Pi^{ND}<em>{2,\Omega} \right) \geq E_0 \left( \Pi^{D}</em>{2,\Omega} \right)$</td>
</tr>
</tbody>
</table>

- **$\mathcal{E}^ND_4$ and $\mathcal{E}^D_4$**

Using the information of the relevant tables, we notice that

- the expected profits of informed traders at $t = 2$ corresponding to $\mathcal{E}^ND_4$ are equal to the lowest expected profits of informed traders at $t = 2$ corresponding to $\mathcal{E}^D_4$ (the expected profits in the case $I_1$). Therefore, we conclude that $E_0 \left( \Pi^{ND}_{2,\Omega} \right) \leq E_0 \left( \Pi^{D}_{2,\Omega} \right)$.

- the expected profits of uninformed traders at $t = 2$ corresponding to $\mathcal{E}^ND_4$ are lower than the expected profits of uninformed traders at $t = 2$ corresponding to $\mathcal{E}^D_4$ in all the possible cases whenever $\theta^U_2 > 0$. Therefore, we conclude that $E_0 \left( \Pi^{ND}_{2,\Omega} \right) < E_0 \left( \Pi^{D}_{2,\Omega} \right)$.

- **$\mathcal{E}^ND_4$ and $\mathcal{E}^D_5$**

Using the information of the relevant tables, we notice that

- the expected profits of informed traders at $t = 2$ corresponding to $\mathcal{E}^ND_4$ are equal to the lowest expected profits of informed traders at $t = 2$ corresponding to $\mathcal{E}^D_5$ (the expected profits in the case $I_1$). Given that the migration from $\mathcal{E}^ND_4$ to $\mathcal{E}^D_5$ has only sense in case $I_6$, we conclude that $E_0 \left( \Pi^{ND}_{2,\Omega} \right) < E_0 \left( \Pi^{D}_{2,\Omega} \right)$.

- the comparison between expected profits of uninformed traders at $t = 2$ corresponding to $\mathcal{E}^ND_4$ and the expected profits of uninformed traders at $t = 2$ corresponding to $\mathcal{E}^D_5$ is in general ambiguous. Direct computations yield that

<table>
<thead>
<tr>
<th>Condition</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta^U_2 &gt; \frac{2\lambda(\pi_2 - k_1)}{(1-\lambda)(k_2-k_1) + \lambda(1-\pi)}$</td>
<td>$E_0 \left( \Pi^{ND}<em>{2,\Omega} \right) &lt; E_0 \left( \Pi^{D}</em>{2,\Omega} \right)$</td>
</tr>
<tr>
<td>$\theta^U_2 \leq \frac{2\lambda(\pi_2 - k_1)}{(1-\lambda)(k_2-k_1) + \lambda(1-\pi)}$</td>
<td>$E_0 \left( \Pi^{ND}<em>{2,\Omega} \right) \geq E_0 \left( \Pi^{D}</em>{2,\Omega} \right)$</td>
</tr>
</tbody>
</table>

- **$\mathcal{E}^ND_4$ and $\mathcal{E}^D_6$**

Using the information of the relevant tables, we notice that

- the expected profits of informed traders at $t = 2$ corresponding to $\mathcal{E}^ND_4$ are equal to the lowest expected profits of informed traders at $t = 2$ corresponding to $\mathcal{E}^D_6$ (the expected profits in the case $I_1$). Therefore, we conclude that $E_0 \left( \Pi^{ND}_{2,\Omega} \right) \leq E_0 \left( \Pi^{D}_{2,\Omega} \right)$.

- the comparison between expected profits of uninformed traders at $t = 2$ corresponding to $\mathcal{E}^ND_4$ and the expected profits of uninformed traders at $t = 2$ corresponding to $\mathcal{E}^D_6$ is in general ambiguous. Direct computations yield that
\[
\frac{\theta_j^U}{2} > \frac{2\lambda \pi (\kappa - k_1)}{(1 - \lambda)(k_2 - k_1)} \\
E_0 \left( \Pi_{2,U}^{e^{\text{IN}}} \right) < E_0 \left( \Pi_{2,U}^{e^{\text{DIR}}} \right)
\]
\[
\frac{\theta_j^U}{2} \leq \frac{2\lambda \pi (\kappa - k_1)}{(1 - \lambda)(k_2 - k_1)} \\
E_0 \left( \Pi_{2,U}^{e^{\text{IN}}} \right) \geq E_0 \left( \Pi_{2,U}^{e^{\text{DIR}}} \right)
\]

Finally, we sum up the results obtained in this part for \( t = 2 \):

### Informed Trader

<table>
<thead>
<tr>
<th>( \varepsilon_{1}^D )</th>
<th>( \varepsilon_{2}^D )</th>
<th>( \varepsilon_{3}^D )</th>
<th>( \varepsilon_{4}^D )</th>
<th>( \varepsilon_{5}^D )</th>
<th>( \varepsilon_{6}^D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_{1}^{ND} )</td>
<td>≤</td>
<td>&lt;</td>
<td>&lt;</td>
<td>( \varepsilon_{4}^D )</td>
<td>( \varepsilon_{5}^D )</td>
</tr>
<tr>
<td>( \varepsilon_{2}^{ND} )</td>
<td>≤</td>
<td>&lt;</td>
<td>&lt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_{3}^{ND} )</td>
<td>≤</td>
<td>≤</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_{4}^{ND} )</td>
<td>≤</td>
<td>&lt;</td>
<td>≤</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Uninformed Trader

<table>
<thead>
<tr>
<th>( \varepsilon_{1}^D )</th>
<th>( \varepsilon_{2}^D )</th>
<th>( \varepsilon_{3}^D )</th>
<th>( \varepsilon_{4}^D )</th>
<th>( \varepsilon_{5}^D )</th>
<th>( \varepsilon_{6}^D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_{1}^{ND} )</td>
<td>&lt;</td>
<td>&lt;</td>
<td>( \varepsilon_{4}^D )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_{2}^{ND} )</td>
<td>&lt;</td>
<td>≤</td>
<td>( \varepsilon_{3}^D )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_{3}^{ND} )</td>
<td>&lt;</td>
<td>&lt;</td>
<td>( \varepsilon_{5}^D )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_{4}^{ND} )</td>
<td>&lt;</td>
<td>&lt;</td>
<td>( \varepsilon_{6}^D )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Internet Appendix IV (Additional Graphs)

In this Appendix we illustrate graphically how different parameter values affect the existence of the equilibria. In Figure 3, in the paper, we depicted the optimal strategies at $t = 1$ in the single-venue market for $k_1 = 5$. In what it follows, we present two additional cases: when the market liquidity is high, $k_1 = 2$, and very illiquid $k_1 = 30$. This last example, with values that are not very realistic is selected to display the four possible equilibria. Remember that in the case $k_1 = 1$ the only possible equilibrium is (BMO, SMO, NT, NT).

Figure IV.1: Optimal strategies at $t = 1$ in the single-venue market. Parameters values: $k_1 = 2$, $\lambda = 0.5$, $\tau = 0.05$, $\delta = 0.95$.

Figure IV.2: Optimal strategies at $t = 1$ in the single-venue market. Parameters values: $k_1 = 30$, $\lambda = 0.9$, $\tau = 0.05$, $\delta = 0.95$. 

93
Figure IV.3 illustrates the optimal strategies at $t = 1$ with respect to the fundamental asset’s volatility and information asymmetry in the case market is very liquid, $k_1 = 2$, and for several values of $\theta_1^I \in \{0.05, 0.19, 0.25, 0.35\}$, which we show in Panels a), b), c), and d), respectively.

Figure IV.3: Optimal strategies at $t = 1$ with dark pool. Parameters values: $k_1 = 2$, $k_2 = 3$, $\lambda = 0.5$, $\tau = 0.05$, and $\delta = 0.95$. In Panel a) $\theta_1^I = 0.05$, in Panel b) $\theta_1^I = 0.19$, in Panel c) $\theta_1^I = 0.25$, and in Panel d) $\theta_1^I = 0.35$.

Figure IV.4 illustrates the optimal strategy profiles at $t = 1$ with respect to the asset’s volatility and the probability of execution for the informed trader in the $DP$ in the first trading period ($\theta^I_1$) in the case market is very liquid, $k_1 = 2$. 
Figure IV.4: Optimal strategies at $t = 1$ with dark pool. Parameters values: $k_1 = 2$, $k_2 = 3$, $\lambda = 0.5$, $\pi = 0.5$, $\tau = 0.05$, and $\delta = 0.95$. 